Homework - EE-I $\boxed{7} \to \boxed{10}$ (Answers)

1. $A = i\frac{d}{dt}$. Need $(Af, g) = (f, Ag)$

$$(Af, g) = \int i \frac{df}{dt} \overline{g(t)}\, dt = i f(t)\overline{g(t)}\Big|_{-\infty}^{\infty} - i \int f(t) \frac{d\overline{g(t)}}{dt} dt$$

$$= \int f(t) \cdot -i \frac{d\overline{g}}{dt} dt$$

$$= \int f(t) \cdot \overline{i\frac{dg}{dt}}\, dt$$

$$= (f, Ag)$$

2. With $Af(t) = f(t-\tau)$, we need

$$(Af, Ag) = \int f(t-\tau)\overline{g(t-\tau)}\, dt$$

$$= \int f(t)\overline{g(t)}\, dt = (f, g).$$

3. $$e^{isL} f(t) = \sum_{k=0}^{\infty} \frac{(is)^k}{k!} L^k f(t) = \sum_{k=0}^{\infty} \frac{(is)^k}{k!} (i)^k \frac{d^k}{dt^k} f(t)$$

$$= \sum (-1)^k \frac{s^k}{k!} \frac{d^k}{dt^k} f(t)$$

$$= \text{Taylor expansion for } f(t-s).$$

Homework $\in$ E+I $\sigma_1 \to \sigma_1$ (Answers)

Notes At least formally, $e^{sL} e^{tL} = e^{(s+t)L}$
via the powerseries def. of e^{sL}

[Only fine point: does the series converge?]

Conceptually, if A is self adjoint then e^{isA} is unitary:

$$(e^{isA} v, e^{isA} u) = (v, e^{-isA} e^{isA} u) = (v, u)$$

term-by-term

$$(iA^k v, u) = (v, -iA^k u)$$

4. $M_\varphi(v) = \frac{(v,\varphi)}{(\varphi,\varphi)} \varphi$

A. Self-adjoint?

$$(M_\varphi(v), w) = \left(\frac{(v,\varphi)}{(\varphi,\varphi)} \varphi, w\right) = \frac{(v,\varphi)(\varphi,w)}{(\varphi,\varphi)}$$

$$(v, M_\varphi(w)) = \left(v, \frac{(w,\varphi)}{(\varphi,\varphi)} \varphi\right) = \left(\overline{\frac{(w,\varphi)}{(\varphi,\varphi)}} v, \varphi\right)$$

$$= \frac{\overline{(w,\varphi)}}{(\varphi,\varphi)} (v,\varphi) = \frac{(\varphi,w)(v,\varphi)}{(\varphi,\varphi)}$$

B. Idempotent? $M_\varphi(M_\varphi v) = \dfrac{\left(\frac{(v,\varphi)}{(\varphi,\varphi)} \varphi, \varphi\right)}{(\varphi,\varphi)} \varphi = \dfrac{\frac{(v,\varphi)(\varphi,\varphi)}{(\varphi,\varphi)}}{(\varphi,\varphi)} \varphi$