

Homework [ (9) to (10) of GF, VS ]

1. Let  $K$  be a field.

Are the  $n \times n$  matrices with elements in  $K$  a field?

2. Let  $K$  be the finite field  $\{0, 1\}$  with  $+$  and  $\cdot$  interpreted mod 2.

Let  $p(x) = x^3 + x + 1$ .  
Does  $p(x)$  factor (in  $K$ )?

Consider the formal sums  $\beta_0 + \beta_1 x + \beta_2 x^2$ ,  
with  $\beta_i$ 's in  $K$ , and  $p(x) \equiv 0$ .

How many distinct sums are there?

Are they closed under  $+$ ,  $\cdot$ ?

Are they closed under inverse?

Hint: write  $1, x, x^2, x^3, x^4, \dots$   
as formal sums.

[ This is the Galois field of size  $2^3$  ]