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Power Spectra HW 2 Ans (15-27)

1. Using the same logic as pp. 16-17,

there is a contribution to $\tilde{\hat{C}}_x(\omega)$ of

$$\frac{1}{\Delta\tau} r(-\tau) * \underbrace{\rho(\tau) * \cdots * \rho(\tau)}_{n \text{ times}} * r(\tau)$$ for the n^{th} spike, given inp-event at 0.

$$② \rightarrow \frac{\lambda\, \hat{\rho}(\omega)\, \hat{r}(\omega)\, \hat{r}(-\omega)}{1 - \hat{\rho}(\omega)}$$

Similarly

$$\tilde{\hat{P}}_r(\omega) = \lambda \left[\hat{r}(0) + \frac{\hat{\rho}(\omega) |\hat{r}(\omega)|^2}{1 - \hat{\rho}(\omega)} + \overline{\left(\frac{\hat{\rho}(\omega) \hat{r}(\omega)}{1 - \hat{\rho}(\omega)} \right)}^2 \right]$$

$$= \lambda \left[\left(\hat{r}(0) - |\hat{r}(\omega)|^2 \right) + \frac{1 - |\hat{\rho}(\omega)|^2}{|1 - \hat{\rho}(\omega)|^2} |\hat{r}(\omega)|^2 \right]$$

2. Let λ = rate of overall process, λ_1 + λ_2 rates of the processes ρ_1 + ρ_2.

① - contrib. is λ.

② - contrib is $\frac{1}{\Delta\tau} \left[\rho_2 + \rho_2 * \rho_1 + \rho_2 * \rho_1 * \rho_2 + \cdots \right]$

given a ρ_1 - event at $t = 0$

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2, ctd. $\lim_{\Delta\tau \to 0} \frac{1}{\Delta\tau}\left[\rho_1 + \rho_1 * \rho_2 + \rho_1 * \rho_2 * \rho_1 + \cdots\right]$

given a ρ_2-event at $t=0$.

renewal dens: ρ_1 ρ_2 ρ_1 ρ_2 etc.

event type: 2 1 2 1 2

Mean interval length for $\rho_1 = \int t\rho_1(t)\,dt = \frac{1}{\lambda_1}$

Mean overall interval length $= \frac{1}{2}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)$

$$\text{Rate} = \frac{1}{\frac{1}{2}\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)} = \frac{2\lambda_1\lambda_2}{\lambda_1+\lambda_2} = \lambda.$$

$$\text{Prob}(\rho_1\text{-event at } t=0 \text{ to } t+\Delta t) = (\lambda_1 \Delta t)\cdot \frac{\frac{1}{2}\frac{1}{\lambda_1}\ [\rho_1\text{-intervals}]}{\frac{1}{2}\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)\ [\text{all intervals}]}$$

$$= \left(\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\right)\Delta t$$

So: ① ② ②'

$$P(\omega) = \frac{2\lambda_1\lambda_2}{\lambda_1+\lambda_2} + \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\left[\frac{\tilde{\rho}_2(\omega)(1+\tilde{\rho}_1(\omega))}{1-\tilde{\rho}_1(\omega)\tilde{\rho}_2(\omega)} + \frac{\tilde{\rho}_1(\omega)(1+\tilde{\rho}_2(\omega))}{1-\tilde{\rho}_1(\omega)\tilde{\rho}_2(\omega)}\right] + \text{c.c.}$$

$$= \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\left[\frac{1}{1-\tilde{\rho}_1(\omega)\tilde{\rho}_2(\omega)}\right]\left[(1+\tilde{\rho}_1(\omega))(1+\tilde{\rho}_2(\omega))\right] + \text{c.c.}$$

③ Power Spectra HW 2 Ans (15-27)

3. Use AR approach.

$$y_n = X_n + a_1 y_{n-1}$$

Yule-Walker "equations" are

$$a_1 \langle y_{n-1}^2 \rangle = \langle y_n y_{n-1} \rangle$$

$$a_1 = \rho.$$

$$\langle X_n^2 \rangle = \langle (y_n - a_1 y_{n-1})^2 \rangle = \langle y_n^2 + a^2 y_{n-1}^2 - 2a y_n y_{n-1} \rangle$$
$$= 1 + \rho^2 - 2\rho^2$$
$$= 1 - \rho^2.$$

$$P_x(\omega) = \frac{1-\rho^2}{|1-\rho e^{i\omega}|^2}.$$

<u>For cos</u>:

$$y_n = X_n + a_1 y_{n-1} = X_n + a_1 (X_{n-1} + a y_{n-2})$$

so

$$\langle y_n y_{n-2} \rangle = \rho^2$$

& similarly $\langle y_n y_{n-k} \rangle = \rho^{|k|}$.