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Power Spectra HW 2 Ans (15-27)

1. Using the same logic as pp. 16-17,

there is a contrib'n to $\hat{C}_x(\omega)$ of

$$\frac{1}{\Delta T} r(-T) * \underbrace{\rho(\uparrow) * \dots * \rho(\uparrow)}_{n \text{ times}} * r(T) \quad \text{for the } n^{\text{th}} \text{ spike, given dep-event at } 0.$$

$$\textcircled{2} \rightarrow \frac{\mathcal{L} \{ \hat{p}(\omega) \hat{r}(\omega) \hat{r}^*(-\omega) \}}{1 - \hat{p}(\omega)}$$

Similarly

$$\hat{S}_r(\omega) = \mathcal{L} \left[\hat{r}(0) + \frac{\hat{p}(\omega) |\hat{r}(\omega)|^2}{1 - \hat{p}(\omega)} + \frac{(\hat{p}(\omega) \hat{r}(\omega))^2}{(1 - \hat{p}(\omega))^2} \right]$$

$$= \mathcal{L} \left[(|\hat{r}(0)| - |\hat{r}(\omega)|)^2 + \frac{1 - |\hat{p}(\omega)|^2}{|1 - \hat{p}(\omega)|^2} |\hat{r}(\omega)|^2 \right]$$

2. Let λ = rate of overall process, λ_1 , λ_2 rates of the processes P_1 , P_2 .① - contrib. is λ .

$$\textcircled{2} - \text{contrib. is } \frac{1}{\Delta T} [p_2^2 + p_2 * p_1 + p_2 * p_1 * p_2 + \dots]$$

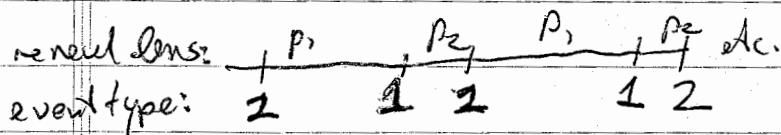
given a p_1 -event at $t=0$

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z.s. also $\frac{1}{\Delta T} [p_1 + p_1 * p_2 + p_1 * p_2 * p_1 + \dots]$

given a p_2 -event at $t=0$.



Mean interval length for $p_1 = \int t p_1(t) dt = \frac{1}{\lambda_1}$

Mean overall interval length = $\frac{1}{2} (\frac{1}{\lambda_1} + \frac{1}{\lambda_2})$

Rate = $\frac{1}{\frac{1}{2} (\frac{1}{\lambda_1} + \frac{1}{\lambda_2})} = \frac{2\lambda_1\lambda_2}{\lambda_1 + \lambda_2} = \lambda$

Prob (p_1 -event at $t=0$) = $(\lambda_1 \Delta t) \cdot \frac{\frac{1}{2} \frac{1}{\lambda_1} [p_1\text{-intervals}]}{\frac{1}{2} (\frac{1}{\lambda_1} + \frac{1}{\lambda_2}) [all\ intervals]}$

$= \left(\frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} \right) \Delta t$

So: ①

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③'

$P(\omega) = \frac{2\lambda_1\lambda_2}{\lambda_1 + \lambda_2} + \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2} \left[\frac{\tilde{p}_1(\omega)(1 + \tilde{p}_1(\omega))}{1 - \tilde{p}_1(\omega)\tilde{p}_2(\omega)} + \frac{\tilde{p}_2(\omega)(1 + \tilde{p}_2(\omega))}{1 - \tilde{p}_1(\omega)\tilde{p}_2(\omega)} \right] + c.c.$

$= \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2} \left[\frac{1}{1 - \tilde{p}_1(\omega)\tilde{p}_2(\omega)} \right] \left[(1 + \tilde{p}_1(\omega))(1 + \tilde{p}_2(\omega)) \right] + c.c.$

③ Power Spectra HW 2 Ans (15-27)

3. Use AR approach.

$$y_n = x_n + a_1 y_{n-1}$$

The-Waller "equations" are

$$a_1 \langle y_{n-1}^2 \rangle = \langle y_n y_{n-1} \rangle$$

$$a_1 = \rho$$

$$\begin{aligned} \langle x_n^2 \rangle &= \langle (y_n - a_1 y_{n-1})^2 \rangle = \langle y_n^2 + a_1^2 y_{n-1}^2 - 2a_1 y_n y_{n-1} \rangle \\ &= 1 + \rho^2 - 2\rho^2 \\ &= 1 - \rho^2 \end{aligned}$$

$$P_x(\omega) = \frac{1 - \rho^2}{|1 - \rho e^{j\omega}|^2}$$

For covs:

$$y_n = x_n + a_1 y_{n-1} = x_n + a_1 (x_{n-1} + a_1 y_{n-2})$$

so

$$\langle y_n y_{n-2} \rangle = \rho^2$$

$$\text{similarly } \langle y_n y_{n-k} \rangle = \rho^{|k|}$$