Algebraic Overview

Homework #1 (2008)

Q1: Eigenvectors of some linear operators in matrix form.
In each case, find the eigenvalues, the eigenvectors, the dimensions of the eigenspaces, and whether a basis can be chosen from the eigenvectors.

A. $A = \begin{pmatrix} q & 1 \\ 0 & q \end{pmatrix}$.

B. $B = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}$ (assume $a > b > c > 0$). Do the eigenvectors form a basis? Hint: Observe that B commutes with $T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, and find the eigenvalues and eigenvectors of T.

C. $C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

D. $D = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

Q2: Adjoints, etc.

A. Work in the vector space of finite dimension N over the complex numbers. Use the standard inner product $\langle x, y \rangle = \sum_{k=1}^{N} x_k \overline{y_k}$ Given an operator A in matrix form (specified by an array a_{kl}, so that if $x = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$, $z = \begin{pmatrix} z_1 \\ \vdots \\ z_N \end{pmatrix}$ and $z = Ax$, then $z_k = \sum_{l=1}^{N} A_{kl} x_l$), find the matrix form of its adjoint A^*.

B. Work in the vector space of complex-valued functions of time, and using the inner product $\langle f, g \rangle = \int_{-\infty}^{\infty} f(t)\overline{g(t)}dt$. Find the adjoint of the time-translation operator $(D_T f)(t) = f(t+T)$.

C. Set up as in B. Find the adjoint of the linear operator *A*, where *Af* is defined by

$$(Af)(t) = \int_{-\infty}^{\infty} A(t,\tau) f(\tau) d\tau \ .$$