Multivariate Analysis

Homework #2 (2008)

This is a simple example of ICA applied to the cocktail party problem. We will generate a mixture of signals and then attempt to unmix them.

Q1. First, let's generate the signals. We take two non-Gaussian sources, each of which can assume a value of -1 or 1 with a probability of 0.5. That is, a "typical" example of four samples is represented by the matrix $S = \begin{pmatrix} -1 & -1 \\ -1 & 1 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$. (Geometrically, these are positioned at the four corners of a square.) Our measured time series are represented by the columns of $Y = SM$. To keep things really simple, we'll assume that the mixing matrix *M* is the identity, $M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, so $Y = S$.

Now, (forgetting that $Y = S$) we want to try to unmix *Y* to recover the sources *S*. We first apply principal components to write $Y = XA$, and then we will look for choices of *A* that maximize the non-Guassian-ness of the columns $X = YA^{-1}$

That is, we look for the eigenvectors and eigenvalues of $Y^TY = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$. This is the identity, so any pair of orthonormal vectors can serve for *A*. So we can write $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, and $A^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

Given this setup, what rotations (i.e., what values of θ) maximize the kurtosis-squared of the columns of YA^{-1}? What rotations maximize the negentropy (minimize the entropy) of the columns of YA^{-1}?