

Nonlinear Systems Theory

Homework #1 (2008)

Laguerre Polynomials. These are classically defined as the orthonormal polynomials with respect to the weight $\exp(-x)$ for $x \geq 0$. Here we calculate orthogonal (not necessarily orthonormal) polynomials with respect to a scaled version of that weight, namely, $W(x) = \frac{1}{b} \exp(-x/b)$ for $x \geq 0$.

Q1: Find an expression for the moments, $M_n = \int_0^{\infty} x^n \left(\frac{1}{b} e^{-x/b} \right) dx$, for $n \leq 5$ (or, in general).

Q2: Carry out the Gram-Schmidt procedure, for the polynomials x^0, x^1, x^2, x^3 , with an inner product defined by the Laguerre weight, $\langle f, g \rangle = \int_0^{\infty} f(x)g(x) \left(\frac{1}{b} e^{-x/b} \right) dx$.