Nonlinear Systems Theory

Homework #2 (2008)

How does the Wiener representation depend on input power?

Recall that for a linear-nonlinear-linear sandwich L_1NL_2, where N is characterized by an input-output relationship f, the nth Wiener kernel is given by

$K_n(t_1,t_2,...,t_n)=c_n\int L_1(t_1-\tau)L_1(t_2-\tau)...L_1(t_n-\tau)L_2(\tau)d\tau$, where c_n is the nth coefficient in the orthogonal expansion of f with respect to Hermite polynomials based on a Gaussian whose variance P is the variance of the signal that emerges from L_1 That is,

$c_n=\frac{1}{n!P^n}\int\limits_{-\infty}^{\infty} f(x)h_n(x;P)Gau(x,P)dx$, where $Gau(x,P)=\frac{1}{\sqrt{2\pi P}}e^{-x^2/2P}$ and the Hermite polynomials $h_n(x;P)$ have the generating function $\sum_{n=0}^{\infty}\frac{z^n}{n!}h_n(x;P)=e^{xz-z^2/2P}$. For example, $h_0=1$, $h_1(x;P)=x$, $h_2(x;P)=x^2-P$, $h_3(x;P)=x^3-3Px$, $h_4(x;P)=x^4-6Px^2+3P^2$, ….

Given this setup, determine how c_n depends on the power in the input signal. In particular, show that

$$\frac{\partial c_n}{\partial P}=\frac{(n+2)(n+1)}{2}c_{n+2}\ .$$

Hint: It will be useful to rewrite the Hermite polynomials in a manner that explicitly recognizes how they scale with P, namely, $h_n(x;P)=P^{n/2}H_n(\frac{x}{\sqrt{P}})$, where

$$\sum_{n=0}^{\infty}\frac{z^n}{n!}H_n(u)=e^{uz-z^2/2}\ .$$