Exam, 2008-2009

Do any three questions, or, any two questions provided that one of them is Q2 or Q3.

Q1. Power spectrum of synaptic shots.

Consider a model synaptic potential (sometimes called an "alpha-function"), namely

$$G(t;G_0,n,\tau_a,\tau_b) = G_0 \frac{t^n}{n!}\left(\frac{1}{\tau_a^{n+1}}e^{-t/\tau_a} - \frac{1}{\tau_b^{n+1}}e^{-t/\tau_b}\right),$$ and assume that these events occur in a

Poisson fashion, at rate λ. What is the power spectrum of the resulting process?

Q2. How does the Wiener representation depend on input mean?
Recall that for a linear-nonlinear-linear sandwich L_1NL_2, where N is characterized by an input-output relationship f, the nth Wiener kernel is given by

$$K_n(t_1,t_2,...,t_n) = c_n\int L_1(t_1-\tau)L_1(t_2-\tau)...L_1(t_n-\tau)L_2(\tau)d\tau,$$ where c_n is the nth coefficient

in the orthogonal expansion of f with respect to Hermite polynomials based on the Gaussian that emerges from from L_1. Determine how c_n depends on the mean of the input signal.

Q3. Relationships among m-sequences.
The polynomial $x^6 = x+1$ generates a finite (Galois) field of size 2^6 and, consequently, an m-sequence of length 63. Recall that an m-sequence can be created by reading down a column of the table of powers of x. The first portion of this table is presented here:

k	x^k:	coef of x^5	coef of x^4	coef of x^3	coef of x^2	coef of x^1	coef of x^0
0		0	0	0	0	0	1
1		0	0	0	0	1	0
2		0	0	0	1	0	0
3		0	0	1	0	0	0
4		0	1	0	0	0	0
5		1	0	0	0	0	0
6		0	0	0	0	1	1
7		0	0	0	1	1	0
8		0	0	1	1	0	0
9		0	1	1	0	0	0
10		1	1	0	0	0	0
11		1	0	0	0	1	1
12		0	0	0	1	0	1
13		0	0	1	0	1	0
14		0	1	0	1	0	0

Consider the transformation $y = x^c$. This generates a sequence of coefficients $y^k = x^{ck}$, consisting of taking every c-th coefficient from the original m-sequence.

In answering the questions below, it will be useful to recall the following property of finite fields. Since the nonzero elements form a group of size 63 under multiplication, any nonzero element z satisfies $z^{63} = 1$. (This corresponds to the fact that every nonzero element is some power of x, and $x^{63} = 1$.)

A. Generate the sequence that results for $c=2$. How does it relate to the original m-sequence? Is it an m-sequence? If so, find the corresponding polynomial that generates it.

B. Generate the sequence that results for $c=62$. How does it relate to the original m-sequence? Is it an m-sequence? If so, find the corresponding polynomial that generates it.

C. Generate the sequence that results for $c=7$. How does it relate to the original m-sequence? Is it an m-sequence? If so, find the corresponding polynomial that generates it.

Bonus. D. Generate the sequence that results for $c=5$. How does it relate to the original m-sequence? Is it an m-sequence? If so, find the corresponding polynomial that generates it.

Bonus. E. Develop a strategy to characterize the values of c that lead to the above 4 kinds of behaviors.

Q4: How does calculation of Wiener kernels behave when datasets are pooled?
Setup: You have two measurements of the response, $R^{[1]}(t)$ and $R^{[2]}(t)$, to the same stimulus $s(t)$. Consider estimation of the second-order Wiener kernel $K_2(\tau_1,\tau_2)$ from (a) $R^{[1]}(t)$ alone, (b) $R^{[2]}(t)$ alone, and (c) the average, $\frac{1}{2}\left(R^{[1]}(t)+R^{[2]}(t)\right)$ via cross-correlation (e.g., NLST notes page 32). Call these estimates $K_2^{[1]}(\tau_1,\tau_2)$, $K_2^{[2]}(\tau_1,\tau_2)$, and $K_2^{average}(\tau_1,\tau_2)$.
With this setup,
(A) is there any relationship between $K_2^{[1]}(\tau_1,\tau_2)$, $K_2^{[2]}(\tau_1,\tau_2)$, and $K_2^{average}(\tau_1,\tau_2)$?

(B) Same as (A), but for cross-correlation estimates of the general (nth-order) kernel $K_n(\tau_1,\tau_2,...\tau_n)$.

(C) What, if anything, can one learn from $K_2^{[1]}(\tau_1,\tau_2)$ and $K_2^{[2]}(\tau_1,\tau_2)$ that one cannot learn from $K_2^{average}(\tau_1,\tau_2)$?

Q5. Relationship between regression and principal components analysis.
Consider the following two scenarios (detailed below) involving analysis of multiple datasets. In scenario 1, you first regress, and then apply principal components analysis.

In scenario 2, you first apply principal components analysis, and then regress. Under what circumstances are the results identical? (Justify your answer)

You are given a matrix Y containing multiple datasets, and the mth element of the rth dataset is the matrix element y_{mr}. You are also given a matrix X of n regressors (the mth element of the nth regressor is x_{mn}).

Scenario 1: You carry out one regression for each dataset, finding a matrix A for which a_{nr} is the contribution of the nth regressor to the rth dataset, by minimizing

$$R = \sum_{m,r}(\sum_n x_{mn}a_{nr} - y_{mr})^2 = tr\left((XA-Y)^T(XA-Y)\right). \text{ This yields } Y^{fit} = XA. \text{ You then}$$

calculate the first h principal components of Y^{fit}, and project Y^{fit} onto these principal components, leading to $Y^{fit,PCA}$.

Scenario 2. You calculate the first h principal components of Y, and project Y onto these principal components, leading to Y^{PCA}. You then regress Y^{PCA} on the regressors X, yielding regression coefficients A^{PCA} and $Y^{PCA,fit} = XA^{PCA}$.

Discuss the relationship, if any, between $Y^{fit,PCA}$ and $Y^{PCA,fit}$.

Q6. Composition of linear filters.
Consider the following linear system, with input $S(t)$, output $R(t)$, and linear filters F, G, H, K with transfer functions $F(\omega)$, $G(\omega)$, $H(\omega)$, and $K(\omega)$. Find the transfer function of the combined system.