Linear Transformations and Group Representations

Homework #2 (2010-2011)

Q1: Classifying some operators

In each case, determine whether the operators are normal, self-adjoint, unitary, or projections, using the standard inner product for a finite-dimensional space (A, B, C, D), or for complex-valued functions on the line (E, F, G, H). Note: G and H are a bit harder.

A. $A=\begin{pmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$.

B. $B=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\phi & \sin\varphi\\ 0 & -\sin\varphi & \cos\varphi\end{pmatrix}$.

C. $C=\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & 0 & 0\\ \frac{1}{2} & \frac{1}{2} & 0 & 0\\ 0 & 0 & \frac{1}{2} & \frac{1}{2}\\ 0 & 0 & \frac{1}{2} & \frac{1}{2}\end{pmatrix}$.

D. $D=\begin{pmatrix}0 & qi\\ -qi & 0\end{pmatrix}$, $q$ real.

E. $Tf(x)=\frac{1}{2}\left(f(x)+f(-x)\right)$.

F. $Wf(x)=xf(x)$.

G. $Yf(x)=f'(x)$ ( $f'(x)=\frac{df}{dx}$ ).

H. $Zf(x)=if'(x)$ ( $f'(x)=\frac{df}{dx}$ ).

Q2. Making self-adjoint operators and projections

Part A. For any operator $A$, show $A^*A$ is self-adjoint.
Part B. Assuming that $B^*B$ has an inverse, show
$P_B=B(B^*B)^{-1}B^*$ is a projection, by showing that it is idempotent and self-adjoint.