Linear Systems, Black Boxes, and Beyond

Homework #3 (2012-2013)

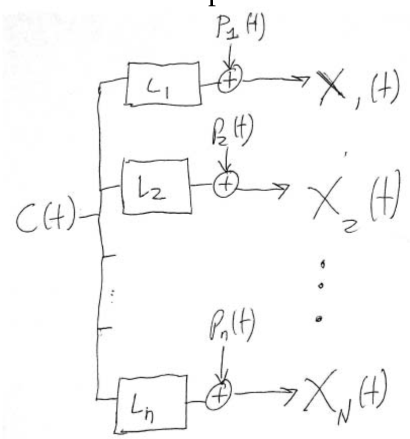
Q1: Spectra of "gamma" renewal processes. A *k*th-order gamma process at rate λ can be constructed by (a) creating a "hidden" Poisson process of rate $k\lambda$, and (b) taking every *k*th spike.

A. Write down the Fourier transform of the renewal density for this process.

B. Write down the power spectrum for this process.

C. Graph the power spectrum for $k = \{1,2,4,8,16,32,64,128\}$.

Q2: A common noise source feeding multiple observed signals. Consider *N* observed signals, $X_i(t)$, constructed as follows: There is a common noise source $C(t)$, and $X_i(t)$ results from the addition of this signal, as filtered by L_i, to a "private" noise source $P_i(t)$. The common noise sources and the private noise sources are all assumed to be independent of each other.



A. Calculate the spectra $P_{X_i}(\omega)$ and the cross-spectra $P_{X_i X_j}(\omega)$ in terms of the spectra of the common noise, $P_C(\omega)$, the private noises $P_{P_i}(\omega)$, and the transfer functions $\tilde{L}_i(\omega)$. The spectra and coherences are the elements of the cross-spectral matrix.

B. In the special case that all of the private noises are 0, calculate the global coherence. As in Cimenser et al. (PNAS 2011, http://www.pnas.org/content/108/21/8832.full), the global coherence is the ratio of the first eigenvector of the cross-spectral matrix, to its trace.

C. Calculate the global coherence under the assumption that common noise source $C(t)$ is 0, and all of the private noise sources have the same power spectrum $P_P(\omega)$.