Linear Systems, Black Boxes, and Beyond

Homework #2 (2014-2015), Questions

Q1: Power spectra of some random processes.

Say an output, $y(t)$, is related to an input, $x(t)$ by $\frac{dy}{dt} = x - ky$. That is, $y$ integrates $x$, but tends to return (decay) to 0 at a rate $k$.

A. Determine the transfer function that relates $y$ to $x$. (Hint – find the response to $x(t) = e^{i\omega t}$).

B. In the above scenario, if $x(t)$ is white noise with power per bandwidth equal to $a$, i.e., $P_X(\omega) = a$, find $P_Y(\omega)$.

C. In the limit that the rate of return is extremely slow (i.e., as $k \to 0$), the above system simply integrates its input. What is its power spectrum?

Q2. Say a system $F$ is a parallel combination of two systems: one component is $2kL$ (where $L$ is as above); the second is system whose response to $x(t)$ is $-x(t)$.

A. What is the transfer function $\hat{F}(\omega)$?

B. Given an input $x(t)$ and an output $y(t)$, how are the power spectra of input and output related?