

Linear Transformations and Group Representations

Homework #2 (2014-2015), Questions

Q1: Properties of self-adjoint and unitary operators

- A. Say A and B are both self-adjoint. Is $A + B$ self-adjoint?
- B. Say A and B are both self-adjoint. Is AB self-adjoint?
- C. Say A and B are both unitary. Is $A + B$ unitary?
- D. Say A and B are both unitary. Is AB unitary?

Q2. Time translation is unitary

Recall that the time translation operator D_T is defined by $(D_T v)(t) = v(t + T)$. Show that D_T is unitary.

Q3. Relationship between unitary and self-adjoint operators

- A. Say A is self-adjoint. Show that $(iA)^* = -(iA)$.
- B. Say A is self-adjoint. Show that $U = e^{iA}$ is unitary. Do this by considering the formal power series definition $e^M = \sum_{j=0}^{\infty} \frac{1}{j!} M^j$.