Linear Transformations and Group Representations

Homework #1 (2018-2019), Questions

Q1: Eigenvectors and eigenvalues of time-translation

A. Consider two vectors c_ω and s_ω defined by $c_\omega(t)=\cos(\omega t)$ and $s_\omega(t)=\sin(\omega t)$, and the vector space V_ω that they span. As before, define $(D_T v)(t)=v(t+T)$. Show that $D_T c_\omega$ and $D_T s_\omega$ are in V_ω by displaying $D_T c_\omega$ and $D_T s_\omega$ as linear combinations of c_ω and s_ω.

B. Express D_T as a 2×2 matrix, using c_ω and s_ω as a basis.

C. Write the characteristic equation for the 2×2 matrix in part B.

D. Solve the characteristic equation in Part C to determine the eigenvalues of D_T in V_ω.

E. Show that $c_\omega \pm i s_\omega$ are eigenvectors of D_T.

Q2: Eigenvectors and eigenvalues of the derivative

A. Setup is the same as Q1, but with the transformation Bv defined by $(Bv)(t)=v'(t)$, i.e., the derivative, rather than D_T. Display Bc_ω and Bs_ω as linear combinations of c_ω and s_ω.

B. Express B as a 2×2 matrix, using c_ω and s_ω as a basis.

C. Write the characteristic equation for the 2×2 matrix in part B.

D. Solve the characteristic equation in Part C to determine the eigenvalues of B in V_ω.

E. Show that $c_\omega \pm i s_\omega$ are eigenvectors of B.