

Groups, Fields, and Vector Spaces

Homework #1 (2020-2021), Questions

Q1: Group or not a group?

Which are groups? If a group, is it commutative? Finite or infinite? If finite, how many elements? If infinite, is it discrete or continuous? If not a group, why not?

- A. Complex numbers, under addition
- B. Complex numbers, under multiplication
- C. The rotations of a regular pentagon into itself, under composition
- D. The rotations and mirror-reflections of a regular pentagon into itself, under composition
- E. The mirror-reflections of a regular pentagon into itself, under composition
- F. The integers $\{0, 1, 2, \dots, N-1\}$ under addition mod N , i.e., $a \circ b = c$ if $a + b - c$ is a multiple of N .
- G. The set of all translations of 3-space, under composition
- H. The set of all rotations of 3-space, under composition
- I. The set of all $N \times N$ matrices with integer entries, under matrix addition
- J. The set of all $N \times N$ matrices with integer entries, under matrix multiplication
- K. The set of all 2×2 matrices with integer entries and determinant 1, under matrix multiplication

Q2. The “center”

The center of a group G , $Z(G)$, is defined as the set of elements c of G that commute with every element of G i.e, for which $g \circ c = c \circ g$.

- A. Show that the center of a group is a group.
- B. For each of the groups in Q1, find the center.