Linear Systems and Black Boxes

Homework #2 (2020-2021), Questions

Q1: Biased random walks

Here we extend the analysis in the notes to a random walk with a directional bias. We allow the particle to move a step of size b to the right or left, but with unequal probability, so that the probability distribution at time $t+\Delta T$ is related to the probability at time t by convolution with $F_{\Delta T}(x)=\left(p_{-}\delta(x-b)+p_{+}\delta(x+b)\right)$, where $p_{-}=\frac{1}{2}(1-c)$ and $p_{+}=\frac{1}{2}(1+c)$.

A. Calculate $\hat{F}_{\Delta T}(\omega)=\int_{-\infty}^{\infty}F_{\Delta T}(x)e^{-i\omega x}dx$

B. Determine how the probability distribution evolves over time T by determining $\left(\hat{F}_{\Delta T}(\omega)\right)^{T/\Delta T}$ in the limit of $\Delta T\to 0$ with (as in the text) $b^2=A\Delta T$ but also $c^2=C\Delta T$.

C. Given a distribution $q_0(x)=\delta(x)$ at time 0, determine the distribution at time T via Fourier synthesis. $\hat{q}_T(\omega)=\hat{F}_T(\omega)\hat{q}(0)=e^{-\omega^2 AT/2-i\omega T\sqrt{AC}}$. The Fourier synthesis for the unbiased walk (in the notes) will be helpful.

Q2: Another biased random walk

In this random walk, the probabilities are equal, but the step sizes are not: So the probability distribution at time $t+\Delta T$ is related to the probability at time t by convolution with $F_{\Delta T}(x)=\frac{1}{2}\left(\delta(x-b_{-})+\delta(x+b_{+})\right)$, where $b_{-}=b-s$,and $b_{+}=b+s$.

A. Calculate $\hat{F}_{\Delta T}(\omega)=\int_{-\infty}^{\infty}F_{\Delta T}(x)e^{-i\omega x}dx$

B. Determine how the probability distribution evolves over time T by determining $\left(\hat{F}_{\Delta T}(\omega)\right)^{T/\Delta T}$ in the limit of $\Delta T\to 0$ with (as in the text) $b^2=A\Delta T$ but also $s=S\Delta T$.

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C. Can this behavior be distinguished from that of the biased random walk in Q1? Why or why not?