Multivariate Methods

Homework #1 (2020-2021), Questions

Here we use Lagrange Multipliers to find maximum-entropy distributions. The common set-up for these problems is the following:

P is a discrete probability distribution on a set of N values $\{x_j\}$: That is, P_i is the probability that a random draw chooses the value x_i. So $P_i \geq 0$ and $\sum_{i=1}^{N} P_i = 1$.

The entropy of a probability distribution $H(P)$ is defined as $H(P) = -\sum_{i=1}^{N} P_i \log P_i$.

Q1: Find the distribution P that maximizes $H(P)$ (subject to the constraint $\sum_{i=1}^{N} P_i = 1$).

Q2: Find the form of the distribution P that maximizes $H(P)$ subject to a constraint on variance, $\sum_{i=1}^{N} P_i x_i^2 = V$ -- you won't be able to solve for the values of the both Lagrange multipliers, but you can get close.