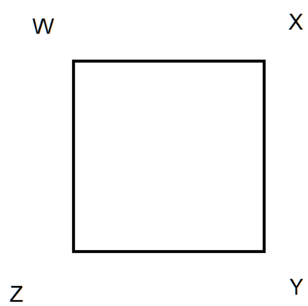
Groups, Fields, and Vector Spaces

Homework #1 (2022-2023), Questions

Q1: Permutations and parity: examples

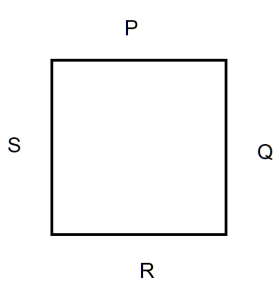
Recall standard permutation notation: Every permutation can be broken down into disjoint cycles, by following around repeated application of the permutation to one element. The permutation that maps A to B, B to C, and C to A is written (ABC) (or, equivalently, (BCA) or (CAB)). The permutation that maps Q to R and R to Q is written (QR) or (RQ). The combination of the two is written, for example, $(ABC)(QR)$.

A. What is the parity of a permutation that has a single cycle of n elements?
B. Consider a square with vertices labeled W, X, Y, and Z in clockwise order.



B1. Display the rotations of the square by $\pi / 2$ as permutations. What is the parity?
B2. Display the rotation of the square by π as a permutation. What is the parity?
B3. Display the reflections that exchange one pair of opposite edges as permutations on the vertices. What is the parity?
B4. Display the reflections that exchange one pair of opposite corners as permutations on the vertices. What is the parity?
(WY) and (XZ), odd parity (one pair swap).

C1-4. Now label the square's edges. Repeat the above, displaying the different kinds of group elements as permutations on the edges.



C1. Rotations of the square by $\pi / 2$:
C2. Rotation of the square by π:
C3. Exchange one pair of opposite edges:
C4. Exchange one pair of opposite corners:

Q2: Permutations, parity, and homomorphisms: an application

Say G is a finite group and $g \in G$ has a permutation representation with odd parity. Show that there is no element h in G for which $g = h^2$.