Groups, Fields, and Vector Spaces

Homework #3 (2022-2023), Questions

Q1. The mapping from linear transformations in V to linear transformations in $V^{\otimes n}$, $sym(V^{\otimes n})$, and $anti(V^{\otimes n})$ preserves composition of operators. But does it preserve addition of operators?

Consider two linear transformations A and B in $Hom(V,V)$ and two vectors $v, w \in V$, and the elementary tensor product $v \otimes w$.

A. Write $(A+B)(v \otimes w)$ in terms of elementary tensor products.

B. Write $(A+B)(v \otimes w - v \otimes w)$ in terms of antisymmetrized elementary tensor products, i.e., in terms of elements of $anti(V^{\otimes 2})$.

C. Is $A(v \otimes w) + B(v \otimes w)$ the same as the quantity in Q1A?

D. Is there a simple relationship between Q1A and Q1C if $A = B$?