

## Linear Transformations and Group Representations

### Homework #2 (2022-2023), Questions

See also Q1 and Q3 of 2021-2022 LTGR homework #2

#### Projections and their relationships

##### Q1: Commuting projections

A. Given projections  $P$  and  $Q$  with  $PQ = 0$ , is  $QP = 0$ ?

Hint: First show that if  $\langle x, z \rangle = 0$  for all  $x$ , then  $z = 0$ . Then consider  $\langle PQx, y \rangle$ .

B. What is the geometric interpretation of this?

C. Given projections  $P$  and  $Q$  with  $PQ = QP$ , is  $PQ$  a projection?

D. (Converse of C) Given projections  $P$  and  $Q$  with  $PQ$  a projection, is  $PQ = QP$ ?

E. Given projections  $P$ ,  $Q$  that commute, and  $PQ \neq 0$ , consider  $X = P - PQ$ ,  $Y = Q - PQ$ , and  $Z = PQ$ :  
(i) Show that  $X$ ,  $Y$ , and  $Z$  are projections. (ii) What is the geometric interpretation?