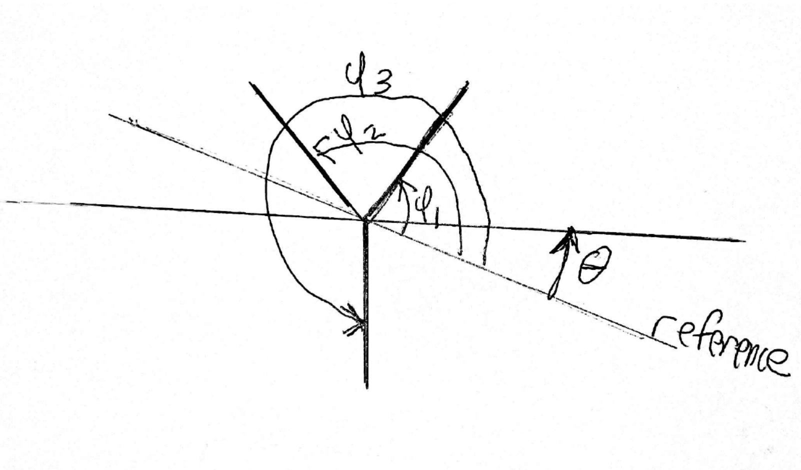
Multivariate Analysis

Homework #2 (2024-2025), Questions

These problems examine the performance of ICA variants in several illustrative "edge cases."

Q1. Moments of a toy distribution

Consider a bivariate distribution $p(\vec{x})$ that is concentrated on $S$ "spokes" emanating from the origin, each of unit length, and is uniformly distributed along those spokes. The directions of the spokes are specified by unit vectors $\vec{v}_k$, $k \in \{1,...,S\}$, each at an angle $\varphi_k$ with respect to some reference direction. Project this distribution onto a line at an angle $\theta$ with respect to the same reference. See diagram below for $S = 3$; $p(\vec{x})$ is concentrated on the solid "Y".




A) Write the $n$th moment $M_n(\theta)$ of the resulting distribution in terms of the $\varphi_k$. B) Under what conditions on the $\vec{v}_k$ are the means of all those distributions is zero, i.e., that $M_1(\theta) = 0$ for all $\theta$? C) What can one say about the shape of $M_2(\theta)$, i.e., about the directions $\theta$ for which $M_2(\theta)$ is maximized or minimized?

Q2. Consider a special case of the distribution in Q1, of four equally-spaced, equally-weighted spokes, with one spoke at an angle of 0 to the reference line. Compute $M_2(\theta)$, $M_3(\theta)$ and $M_4(\theta)$, and determine their maxima and minima.

Q3. Consider a special case of the distribution in Q1, of three equally-spaced, equally-weighted spokes, with one spoke at an angle of 0 to the reference line. Compute $M_2(\theta)$, $M_3(\theta)$ and $M_4(\theta)$, and determine their maxima and minima.

Q4. Consider a special case of the distribution in Q1, of five equally-spaced, equally-weighted spokes. What can you say about the behavior of $M_2(\theta)$, $M_3(\theta)$ and $M_4(\theta)$?