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Real-Time, Adaptive, Model-Independent Control of Low-Dimensional Chaotic and Nonchaotic Dynamical Systems

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***Abstract*—Current model-independent control techniques are limited, from a practical standpoint, by their dependence on a precontrol learning stage. Here we develop a model-independent control technique, for chaotic and nonchaotic low-dimensional dynamical systems, that operates in real-time (i.e., it does not require a learning stage). We show that this technique is adaptive to system nonstationarities, robust to noise, and capable of stabilizing higher-order unstable periodic orbits. Because this technique is real-time, adaptive, and model-independent, it is practical for real-world systems.**

***Index Terms*— Adaptive control, chaos, chaos control, model-independent control.**

I. INTRODUCTION

Many real-world systems are essentially "black-boxes," i.e., little or no information exists regarding their governing equations. Traditional model-based control techniques require a system's governing equations, and are thus, ill-suited for the control of such systems. Recent advances in the area of nonlinear dynamics have produced model-independent control techniques which do not require explicit knowledge of a system's underlying equations. These techniques extract all necessary control information from a prerecorded time series and are thus inherently well-suited for the control of black-box systems.

In the seminal work in this area, Ott, Grebogi, and Yorke (OGY) [1] developed a model-independent control technique for chaotic systems. The OGY control technique utilizes the fact that there are an infinite number of unstable periodic orbits (UPO's) embedded within a chaotic attractor. This technique makes small time-dependent perturbations to an accessible system parameter which repeatedly direct the system's state point toward a desired UPO. The parameter perturbations constrain the system's state point within the neighborhood of a given UPO by exploiting dynamics inherent to that UPO. The OGY technique is practical from an experimental standpoint because it requires no analytical model of the system—all necessary dynamics are estimated from past observations of the system.

Current model-independent control techniques for chaotic [1]–[10] and nonchaotic [11] dynamical systems are applicable to black-box systems because they are able to estimate all necessary system parameters from a prerecorded time series. The flexibility of model-independence in current dynamical control techniques, however, does not come without limitations. The precontrol time-series recording and the corresponding system-dynamics estimation comprise a "learning" stage. For some real-world systems (e.g., cardiac arrhythmias), unwanted dynamics must be eliminated quickly, and thus, the time required for a learning stage may be unavailable. In this brief, we develop a real-time, adaptive, model-independent (RTAMI) control technique for chaotic and nonchaotic systems. The RTAMI technique

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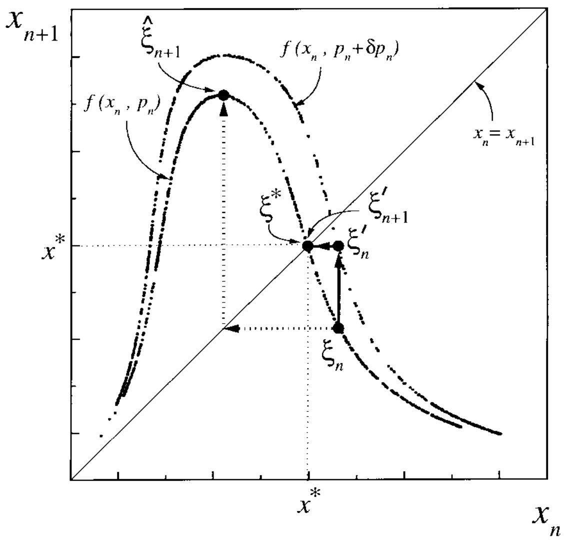

Fig. 1. First-return map of a system described by (1) showing that the ideal δp_n (3) shifts the map from $f(x_n, p_n)$ to $f(x_n, p_n + \delta p_n)$ such that the next system state point is forced to $\xi'_{n+1} = \xi^*$ rather than its expected position $\hat{\xi}_{n+1}$.

requires no learning stage and is thus more appropriate (than current model-independent control techniques) for real-world systems that cannot afford the time for precontrol analysis.

II. The RTAMI Control Technique

The RTAMI control technique is designed to stabilize the flip-saddle unstable periodic fixed point $\xi^* = [x^*, x^*]^T$ (where superscript T denotes transpose and $[x^*, x^*]^T$ is a 2×1 column vector) of a system that can be described effectively by a unimodal one-dimensional map

$$x_{n+1} = f(x_n, p_n) \tag{1}$$

where x_n is the current value (scalar) of one measurable system variable, x_{n+1} is the next value of the same variable, and p_n is the value (scalar) of an accessible system parameter p at index n. The control technique perturbs p such that

$$p_n = \overline{p} + \delta p_n \tag{2}$$

where $\overline{p}$ is the mean parameter value, and δp_n is a perturbation [3], [12], [13], [19]–[21] given by

$$\delta p_n = \frac{x_n - x_n^*}{g_n} \tag{3}$$

where x_n^* is the current estimate of x^*, and g_n is the control sensitivity g at index n. The ideal value of g is the sensitivity of x^* to perturbations: $g_{\text{ideal}} = \delta x^* / \delta p$. As described in [14], control can be achieved for a range of control sensitivities $|g|_{\min} \le |g| \le |g|_{\max}$. As shown in Fig. 1, without any perturbation (i.e., $\delta p_n = 0$), the current state point ξ_n would move to $\hat{\xi}_{n+1}$ (via the dotted arrow). However, the control perturbation of (3) shifts $f(x_n, p_n)$ to $f(x_n, p_n + \delta p_n)$ such that x_n maps to $x'_{n+1} = x^*$ instead of $\hat{x}_{n+1}$. On the first-return map, this shift appears as the movement of ξ_n to ξ'_n (via the solid vertical arrow in Fig. 1). When the map is returned to $f(x_n, p_n)$ for the next iteration, the next state point will be $\xi'_{n+1} \approx \xi^*$, as desired for control.

Learning-stage dependent techniques use static values for x^* and/or g, as estimated from a precontrol time-series recording. In contrast, the RTAMI technique repeatedly estimates x^* and g. In addition to eliminating the need for a learning stage, this adaptability allows for the control of nonstationary systems, which are characterized by parameters that drift over time. When control is initiated, g can be set to an arbitrary value (with the restriction that the sign of g must match that of g_{ideal}). After each measurement of x_n, x^* is estimated using [22]

$$x_n^* = \sum_{i=0}^{N-1} \frac{x_{n-i}}{N}. \tag{4}$$

Equation (4) converges to x^* because consecutive x_n alternate on either side of x^* due to the flip-saddle nature of ξ^*.

At each iteration, after x^* is reestimated via (4), the RTAMI technique evaluates whether the estimate of g should be adapted. The value of g is not adapted if the desired control precision ϵ (which should be slightly larger than the standard deviation of the system noise) has been achieved. Control precision has *not* been achieved if

$$|x_n - x_{n-1}^*| > \epsilon \tag{5}$$

is satisfied by at least L out of the N previous data points, where x_{n-1}^* is the estimate of x^* that was targeted for a given x_n. The L/N factor is used [instead of a single evaluation of (5)] to reduce the influence of noise and spurious points.

If the desired control precision has not been achieved [i.e., (5) has been satisfied by at least L out of the N previous data points], then the magnitude of g is adapted in accordance with the perturbation dynamics shown in Fig. 2. As shown in Fig. 2, if $g = g_{\text{ideal}}$, then the perturbation moves the state point from its current position ξ_n (×) to ξ^* (■) (as in Fig. 1). If $|g|$ is too large (i.e., δp is too small, which moves the attractor to a position similar to that of the open diamonds in Fig. 2), then the state point moves from its current position ξ_n to a position (⧫) closer to ξ^* than would be expected ($\hat{\xi}_{n+1}$) without a perturbation. If $|g|$ is too small (i.e., δp is too large, which moves the attractor to a position similar to that of the open triangles in Fig. 2), then the state point moves from its current position ξ_n to a position (▲) on the same side of the line of identity. This is in contrast to the expected alternation of consecutive state points on either side of the line of identity due to the flip-saddle nature of ξ^*. The criterion

$$\operatorname{sign}(x_n - x_{n-1}) = \operatorname{sign}(x_{n-1} - x_{n-2}) \tag{6}$$

is satisfied when two consecutive state points ($[x_{n-1}, x_{n-2}]$ and $[x_n, x_{n-1}]$) lie on the same side of the line of identity. The RTAMI technique increases the magnitude of g (i.e., $g_{n+1} = g_n \rho$, where ρ is the adjustment factor) if (6) is satisfied for at least L out of the N previous data points. As with the evaluation of control precision (5), the L/N factor is used [instead of a single evaluation of (6)] to reduce the influence of noise and spurious points.

If the magnitude of g is not increased [as dictated by (6)], then the magnitude of g is decreased (i.e., $g_{n+1} = g_n / \rho$) if

$$\frac{1}{N} \sum_{i=0}^{N-1} \frac{|x_{n-i-1} - x_{n-i-2}^*| - |x_{n-i} - x_{n-i-1}^*|}{|x_{n-i-1} - x_{n-i-2}^*|} < r\%. \tag{7}$$

Equation (7) is satisfied if, on average, the distance $|x_{n-i} - x_{n-i-1}^*|$ between a given data point x_{n-i} and its corresponding fixed-point estimate x_{n-i-1}^* is not at least r% smaller than the distance $|x_{n-i-1} - x_{n-i-2}^*|$ between the previous data point x_{n-i-1} and the previous fixed-point estimate x_{n-i-2}^*.

If neither (6) nor (7) is satisfied, then g is not adapted because x is properly approaching the estimate of x^*.

III. Application of the RTAMI Control Technique

To test the RTAMI control technique, we considered the Hénon map with an adjustable parameter A_n, as given by

$$x_n = 1.0 - A_n x_{n-1}^2 + B x_{n-2} \tag{8}$$

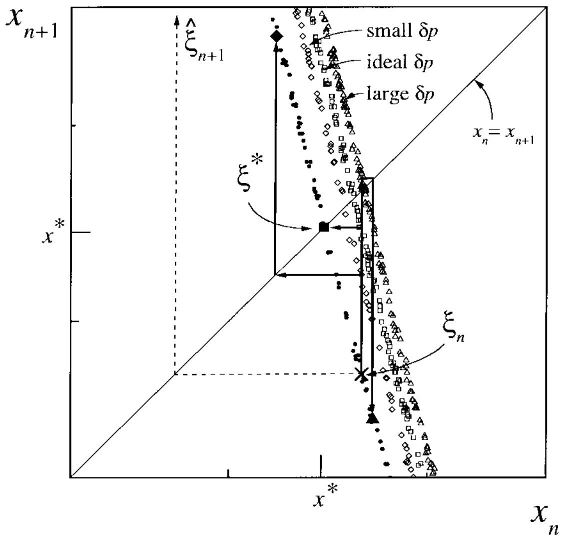

Fig. 2. First-return map of a system described by (1) showing the results of control perturbations that are: ideal (squares), too small (diamonds), and too large (triangles).

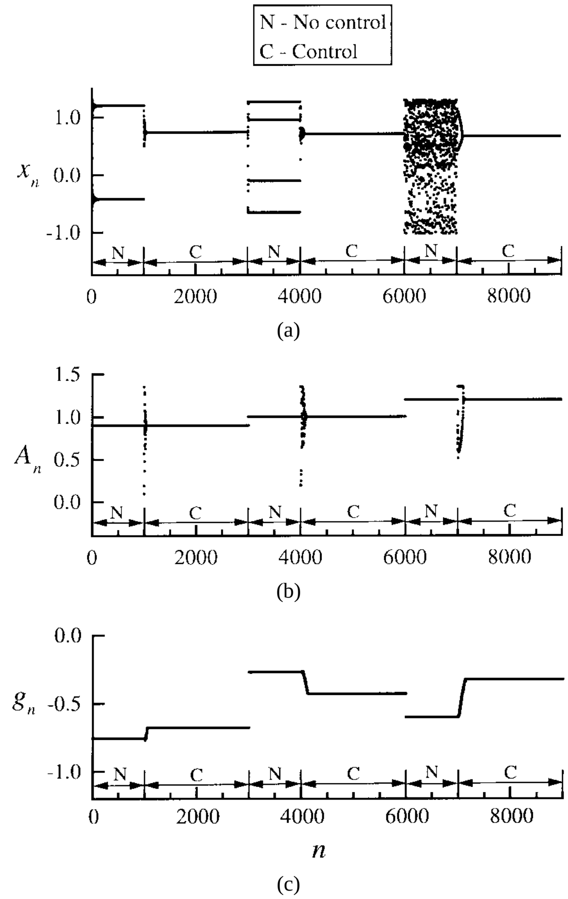

Fig. 3. (a) x_n. (b) A_n. (c) g_n versus n for a representative RTAMI control trial of the Hénon map (8).

where $B = 0.3$. For the control trials, the Hénon-map state point was $\xi_n = [x_n, x_{n+1}]^T$ and the control parameter was A_n (i.e., $p_n \equiv A_n$ in (2) such that $A_n = \overline{A} + \delta A_n$). Fig. 3 shows x_n, A_n, and g_n versus n for a representative Hénon-map control trial (where $N = 10$, $\epsilon = 0.001$, $L = 3$, $r = 5\%$, and $\rho = 1.005$. We found that the RTAMI technique is effective for a wide range of parameter values of N, ϵ, L, r, and ρ.). For $1 \leq n \leq 1000$, $\overline{A}$ was set to 0.90 [Fig. 3(b)], corresponding to a stable precontrol period-2 orbit [Fig. 3(a)]. The initial value for g [$g = -0.76$, Fig. 3(c)] was randomly selected from a uniform distribution $-1.15 \leq g \leq -0.15$. At $n = 1000$, the RTAMI control technique was activated, and the underlying unstable period-1 fixed point was quickly located and stabilized. Initially, the control perturbations were quite large (the perturbations were capped such that $0.10 \leq A \leq 1.35$) because the state point was far from the targeted fixed point. However, as control converged, the perturbations became small ($\delta A_n \approx 10^{-16}$). During control, the magnitude of g was decreased, as dictated by (7), until an appropriate value for g was found. Stabilization was maintained until $n = 3000$, when control was inactivated and $\overline{A}$ was shifted to 1.0, corresponding to a stable period-4 orbit. At $n = 4000$, control was reactivated using another randomly-selected initial value for g ($g = -0.27$), and the new underlying unstable period-1 fixed point was quickly located and stabilized. During control, the magnitude of g was increased, as dictated by (6), until an appropriate value for g was found. Stabilization was maintained until $n = 6000$, when control was inactivated and $\overline{A}$ was shifted to 1.2, corresponding to chaos. At $n = 7000$, control was reactivated using another randomly-selected initial value for g ($g = -0.60$), and the underlying unstable period-1 fixed point of the chaotic system was quickly located and stabilized. During control, the magnitude of g was decreased, as dictated by (7), until an appropriate value for g was found. For each of the three control regions of Fig. 3, the stabilized x^* was within 10^{-6} of the analytically-derived x^*, thereby, demonstrating that the RTAMI technique converged to the real unstable periodic fixed points. Fig. 3 demonstrates that the RTAMI control technique, which does not require a learning stage, is capable of controlling a system "on the fly," as it jumps between different parameter regimes. This is in contrast to current model-independent techniques which require either separate precontrol estimates of a system's dynamics for each parameter regime or nearly-continuous transitions between regimes for tracking.

The RTAMI control technique is inherently robust to noise and outliers because N system values are used in the adaptation of both x_n^* and g, i.e., the effects of spurious values are smoothed via averaging. To demonstrate that the RTAMI technique can be used to control noisy, nonstationary systems, we considered the Hénon map with additive correlated noise, as given by

$$x_n = 1.0 - A_n x_{n-1}^2 + B x_{n-2} + \eta_n \tag{9}$$

where the correlated noise η_n was given by the expression (derived in [15])

$$\eta_n = \eta_{n-1} e^{-(1/\tau_\eta)} + \zeta_n \sqrt{\frac{D_\eta}{\tau_\eta}\left[1 - e^{-(2/\tau_\eta)}\right]} \tag{10}$$

where $D_\eta = 1.0$, $\tau_\eta = 200$, and ζ_n is Gaussian white noise with zero mean and unity standard deviation. Fig. 4 shows x_n, A_n, and g_n versus n for a representative control trial (where $N = 10$, $\epsilon = 0.003$, $L = 3$, $r = 5\%$, and $\rho = 1.005$) of the Hénon map with additive correlated noise. For $1 \leq n \leq 1000$, $\overline{A}$ was set to 1.2, corresponding to chaos. At $n = 1000$, the RTAMI control technique was activated with a randomly selected initial value for g ($g = -0.81$). Stabilization of the underlying unstable period-1 fixed point was quickly achieved. The inset in Fig. 4(a) shows the correlated fluctuations (introduced by η_n) in the stabilized fixed point. This inset shows that the RTAMI control technique is capable of tracking the movement of a nonstationary unstable periodic fixed

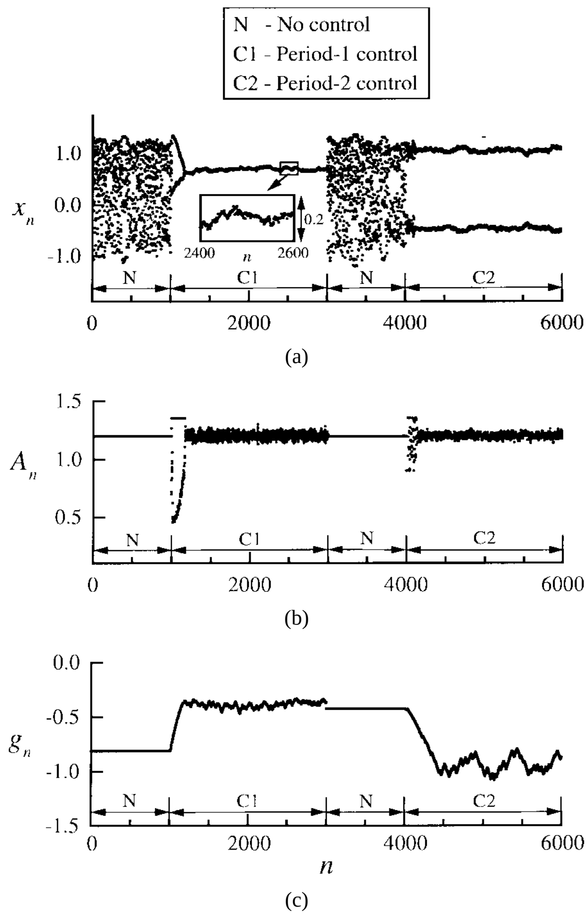

Fig. 4. (a) x_n. (b) A_n. (c) g_n versus n for a representative RTAMI control trial of the Hénon map with additive correlated noise (9). A magnified plot of the correlated fluctuations in x_n (for $2400 \leq n \leq 2600$) is shown in the inset in (a).

point. Stabilization was maintained until $n = 3000$, when control was inactivated. At $n = 4000$, stabilization of the system's underlying unstable period-2 fixed point was activated with a randomly-selected initial value for g ($g = 0.42$). Period-2 stabilization was quickly achieved by updating the estimates for x_n^* and g and applying control interventions at every other iterate, rather than at every iterate (as in [2]). To reduce the likelihood of control failure for higher-period orbits, an extended version of the RTAMI technique could apply control interventions at every iteration. For a period-k UPO, such an approach would estimate different x^*'s and g's for each of the k iterations of the UPO.

In addition to the Hénon map, we also used the RTAMI control technique to stabilize UPO's in: 1) the logistic map in its chaotic and nonchaotic regimes; 2) a discrete-time model [16], [17] of a period-2 cardiac arrhythmia known as atrioventricular nodal alternans; and 3) a continuous-time model [18] of the Belousov–Zhabotinsky oscillatory chemical reaction in its chaotic and nonchaotic regimes. The successful control of these varied systems demonstrates the versatility of the RTAMI control technique.

IV. Summary

In this brief, we developed a real-time (i.e., no learning stage), adaptive (i.e., robust to nonstationarities), model-independent control technique that can be used to stabilize underlying UPO's in low-dimensional chaotic and nonchaotic dynamical systems. We showed that this technique is: 1) applicable to a wide range of parameter regimes and initial values for g; 2) capable of on-the-fly control as a system is switched between parameter regimes; 3) robust to additive noise; and 4) capable of stabilizing higher-order UPO's. The RTAMI technique is quite versatile, as we have used it to stabilize UPO's in the Hénon map, the logistic map, a discrete-time model of a physiological system (atrioventricular nodal alternans), and a continuous-time model of an oscillatory chemical reaction (the Belousov–Zhabotinsky reaction). These developments suggest that the RTAMI control technique is practical for real-world systems.

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