

Homework ($E, E \vee I \quad \square \rightarrow \square$)

1. Show that $f \mapsto i \frac{df}{dt}$ is self-adjoint in L^2
(L^2 modified to ensure $\frac{d}{dt}$ exists)

2. Show that $f(t) \mapsto f(t - \tau)$ is unitary

3. For an operator L , define $e^{sL}(f) = \sum_{k=0}^{\infty} \frac{s^k}{k!} L^k f$,
if the limit exists.

With $Lf = i \frac{df}{dt}$, interpret e^{isL} .

4. For φ any vector, show $M_{\varphi}(v) = \frac{(v, \varphi)}{(\varphi, \varphi)} \varphi$ is a projection.