Entropy & Information Ans

1. Uniform distribution from $-b/2$ to $b/2$ is $p(x) = \frac{1}{b}, |x| < \frac{b}{2}$

Variance is $\int_{-b/2}^{b/2} x^2 p(x)\,dx = \frac{1}{b}\frac{x^3}{3}\Big|_{-b/2}^{b/2} = \frac{b^2}{12}$.

Unit variance $\Rightarrow b = \sqrt{12}$.

Diff. entropy $= -\int_{-b/2}^{b/2} \frac{1}{b}\log_2\frac{1}{b}\,dx = \log_2 b$

D.E. (uniform) $= \log_2(\sqrt{12}) \approx 1.79$

D.E. (Gaussian) $= \frac{1}{\ln 2}\left(\frac{1+\ln 2\pi}{2}\right) \approx 2.05$

2. The string can be recoded, without loss, as a sequence of "3"s & "5"s

eg, 5 3 3 3 3 5 3 3 5 3 ...

This is 1 bit/symbol. But each symbol in the recoded string corr. to either 5 or 3 bits in the original string. Typically one transition every 4 symbols;

$\therefore$ entropy $= \frac{1}{4} \cdot$ [one bit]

3. The process Y is determined by X, & vice versa.

$\therefore$ entropy (differential) per symbol of $Y = \frac{1}{\ln 2}\left[\frac{1+\ln 2\pi}{2} + \frac{\ln V}{2}\right]$