Entropy & Information Ans

Uniform distribution from $-\frac{1}{2}$ to $\frac{1}{2}$; \( p(x) = \frac{1}{b} \cdot \text{rect} \cdot \frac{b}{2} \)

Variance is
\[
\int_{-\frac{b}{2}}^{\frac{b}{2}} x^2 \cdot p(x) \, dx = \frac{1}{b} \cdot \frac{b^3}{6} 
= \frac{b^2}{12}.
\]

Unit variance \( \Rightarrow b = \sqrt{12} \).

Diff. entropy = \[-\frac{b}{2} \cdot \frac{1}{b} \int_{-\frac{b}{2}}^{\frac{b}{2}} \log_2 \frac{1}{b} \, dx = \log_2 b \]

D.E. (uniform) = \( \log_2 \left( \sqrt{12} \right) \approx 1.79 \)

D.E. (Gaussian) = \( \frac{1}{\ln 2} \left( \frac{1 + \ln(2\pi)}{2} \right) \approx 2.05 \)

2. The string can be recorded, model has loss, etc, 5 3 8 3 3 5 3 3 5 3 ...

The's 1 bit/symbol. Each symbol in the recorded string costs 1 more than in the original string. Typically one transmits every 4 symbols;

\[ \text{entropy} = \frac{1}{4} \cdot \text{[symbol]} \]

3. The process \( Y \) is determined by \( X \), vice versa.

\[ \text{entropy (determinant) per symbol of } Y = \frac{1}{2} \left[ \frac{1}{2} \ln \left( \frac{1}{2} \right)^2 + \ln V \right] \]