Homework Answers - F.A. Applic (13) - (27)

1

$$\hat{F}(\omega) = \int_0^\infty e^{-i\omega t} \lambda e^{-\lambda t}\, dt$$

$$= \lambda \int_0^\infty e^{-(i\omega+\lambda)t}\, dt$$

$$= \frac{\lambda}{-(i\omega+\lambda)} e^{-(i\omega+\lambda)t} \Big|_0^\infty$$

$$= \frac{\lambda}{-(i\omega+\lambda)} (0 - 1) = \frac{\lambda}{i\omega+\lambda} = \frac{1}{1+i\omega/\lambda}$$

Pole is at $1+i\omega/\lambda = 0$, $\Rightarrow$ $\omega = i\lambda$.

Residue: $\lim_{\omega \to i\lambda} (\omega - i\lambda) \cdot \frac{\lambda}{i\omega+\lambda} = \frac{\lambda}{i} \lim_{\omega \to i\lambda} \frac{i\omega+\lambda}{i\omega+\lambda}$

$= -i\lambda$.

From (*) on R24:

$F(t) = i \sum_{\text{Pos. UHP}} e^{i\omega t} \operatorname{Res}_\omega \hat{F}(\omega)$ Only pole is $\omega = i\lambda$.

$$F(t) = i\, e^{i(i\lambda)t} (-i\lambda) = \lambda e^{-\lambda t}.$$

② Homework Answers - FA Applications (18) - (27)

2. $F_\lambda(\omega) = \dfrac{1}{1+i\omega/\lambda}$

$$\lim_{N\to\infty}\left(\frac{1}{1+i\omega/N\lambda}\right)^N = \lim_{N\to\infty}\left(1+\frac{i\omega}{N\lambda}\right)^{-N}$$

$$= \lim_{N\to\infty}\left[\left(1+\frac{i\omega}{N\lambda}\right)^{-\frac{N\lambda}{i\omega}}\right]^{\frac{i\omega}{\lambda}}$$

$$= e^{-i\omega/\lambda}$$

This is the Fourier transform of a pure delay, i.e.

if $G(t) = \delta\left(t - \frac{1}{\lambda}\right)$ then $\hat{G}(\omega) = e^{-i\omega/\lambda}$.

A concatenation of fast low pass filters is indistinguishable from a delay.