Fourier Analysis Applic, HW ① - ⑦

1. Direct verif. of convolution thm via elementary means [(a)]

For $s_j = \frac{1}{N} \sum_{r=0}^{N-1} p_{j-r} q_r$, and

$\hat{s}_k, \hat{p}_k, \hat{q}_k$ related to $s_j, p_j, q_j$ as

$$\begin{cases} s_j = \sum_{k=0}^{N-1} e^{\frac{2\pi i}{N} kj} \hat{s}_k \\ \hat{s}_k = \sum e^{-\frac{2\pi i}{N} kj} s_j \end{cases}$$

Show $\hat{s}_k = \hat{p}_k \hat{q}_k$.

2. Properties of FT under time translation. [(a)]

Say $r(t) = s(t+\tau)$.

Given $\hat{s}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} s(t) dt$,

Find $\hat{r}(\omega)$ in terms of $\hat{s}(\omega)$

3. Say $q(t) = \frac{d}{dt} s(t)$. Find $\hat{q}(\omega)$ in terms of $\hat{s}(\omega)$ [(c)]

(Assume anything you need about convergence]

4. Moments. Write $\int_{-\infty}^{\infty} t^n s(t) dt$ in terms of $\hat{s}(\omega)$.

(Assume anything you need about convergence]