Homework ⑨-⑩ of GFVS Answers

1. No - inverses may not exist

2. a) only need to consider binomial factors (of deg 1)

$x^3+x+1 = x(x^2+1)+1$ so x is not a factor

$x^3+x+1 = (x+1)(x^2+x)+1$ so $(x+1)$ " " " "

b) There are 2^3 distinct sums $\beta_0+\beta_1 x+\beta_2 x^2$ with $\beta_i \in \{0,1\}$

Closed under + (trivially) + under ·, by reducing:

$x^3+x+1=0 \Rightarrow x^3=-x-1 \Rightarrow x^3=x+1$.

c) Closed under inverses:

	β_0	β_1	β_2	
$1=$	1	0	0	
$x=$	0	1	0	
$x^2=$	0	0	1	
$x^3=$	1	1	0	$x^3 = x+1$
$x^4=$	0	1	1	$x^4 = x^2+x$
$x^5=$	1	1	1	$x^5 = x^3+x^2 = (x+1)+x^2$
$x^6=$	1	0	1	$x^6 = x(x^2+x+1) = x^3+x^2+x = (x+1)+(x^2+x)$
$x^7=$	1	0	$0=x^0$	$x^7 = x^3+x = (x+1)+x = 1$.

$x^7 = x^0$, so no more elements.

$x^u \cdot x^v = x^{u+v \ [\mathrm{mod}\ 7]}$ ∴ closed under inverses.