

Homework [① to ⑤ of Groups, Fields, Vector Spaces]

1. Group Properties. Say G is a group, and H, K are subgroups.

a) Is $H \cap K$ a subgroup? (intersection)

b) Is $H \cup K$ a subgroup? (union)

2. "Normal" subgroups.

Say N is a subgroup of G . N is "normal" if, for all $h \in G$, and all $n \in N$
 $h n h^{-1} \in N$.

[Equivalently, $hN = Nh$, "right coset = left coset"]

a) If G is commutative, show all subgroups are normal.

b) If N_1, N_2 are normal, then $N_1 \cap N_2$ is normal.

c) Say $\varphi: G \rightarrow H$ is a homomorphism, and

K is the kernel of φ . Show K is normal.

d) Show that the inner automorphisms of a group is a normal subgroup of the automorphisms.