Homework \[ \text{[[1] to [3] SGF, VS]} \]

1. Let \( K \) be a field.

   Are the \( n \times n \) matrices with entries in \( K \) a field?

2. Let \( K \) be the finite field \( \mathbb{Z}_p \) with \( + \) and \( \cdot \) interpreted mod \( p \).

   \( \text{Let } p(x) = x^3 + x + 1 \)

   Does \( p(x) \) factor in \( K \)?

   \[
   \text{Consider the formal sum } \beta_0 + \beta_1 x + \beta_2 x^2, \\
   \text{with } \beta_i \text{ in } K, \text{ and } p(\beta) \equiv 0. \\
   \text{How many distinct sums are there?} \\
   \text{Are they closed under } +, \cdot ? \\
   \text{Are they closed under inversion?} \\
   \text{Hint: Write } 1, x, x^2, x^3, x^4, \ldots \\
   \text{and form sums.} \\
   \text{[This is the finite field of size } 2^3] \\
   \]