

Groups, Fields, and Vector Spaces

Homework #1 (2008) for pages 1-4 of notes

Q1: Display two distinct groups with 4 elements.

Q2: Determine whether the following are groups, and if so, characterize them as: commutative or not commutative, finite or infinite, discrete vs. continuous.

A.. The 2×2 matrices with integer entries and determinant 1, with the operation of matrix multiplication.

B. Real-valued functions $f(t)$, under the operation $+$, defined by $(f + g)(t) = f(t) + g(t)$

C. Real-valued functions $f(t)$, under the operation $*$, defined by

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - z)g(z)dz$$

(This operation is called "convolution")

Q3: (bonus) Find the non-commutative group with the fewest elements.

Q4: (super-bonus) Find all of the groups with 8 elements, and find uses for (i.e., describe) them.