Linear Systems Theory

Homework #1 (2008)

Q1. Some basic properties of Fourier transforms pairs,

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \tag{1}$$

and

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(\omega)e^{i\omega t}d\omega\,. \tag{2}$$

A. Let $g(t) = e^{iat} f(t)$. Find $\tilde{g}(\omega)$ in terms of $\tilde{f}(\omega)$.

B. Let $g(t) = kf(kt)$. Find $\tilde{g}(\omega)$ in terms of $\tilde{f}(\omega)$.

C. Let $g(t) = \dfrac{df(t)}{dt}$. Find $\tilde{g}(\omega)$ in terms of $\tilde{f}(\omega)$.

D. Find $\int_{-\infty}^{\infty} f(t)dt$ in terms of $\tilde{f}(\omega)$.

E. Find $\int_{-\infty}^{\infty} t^m f(t)dt$ in terms of $\tilde{f}(\omega)$.

F. Show that if $f(t) = f(-t)$, then $\tilde{f}(\omega)$ is real.

Q2: Smoothing and averaging filters

Here, we view a smoothing and averaging filter F as a linear transformation on unprocessed signals $s(t)$, to produce a processed signal $r(t) = [F(s)](t)$. But since the transformation can be applied after all data are collected, the "impulse response" function $f(t)$ need not be causal. That is,

$$r(t) = \int_{-\infty}^{\infty} f(\tau)s(t-\tau)d\tau \tag{3}$$

where $f(t)$ can be nonzero for both negative and positive times. There is no change in how the transfer function is defined, namely,

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \tag{4}$$

A. Boxcar average. Define $f_{boxcar}(t) = \begin{cases} \frac{1}{L}, |t| \le L/2 \\ 0, otherwise \end{cases}$. This replaces *s* by its average over a window of length *L*. Find the corresponding transfer function $\tilde{f}_{boxcar}(\omega)$.

B. Triangular average. Define $f_{triangle}(t) = \begin{cases} \frac{(1-|t|/L)}{L}, |t| \le L \\ 0, otherwise \end{cases}$. This replaces *s* by an average over a window of length *L* but weights the central values more heavily. Find the corresponding transfer function $\tilde{f}_{triangle}(\omega)$. Relate the answer to part A.

C. Cosine bell. $f_{bell}(t) = \begin{cases} \frac{1+\cos(\pi t/L)}{2L}, |t| \le L \\ 0, otherwise \end{cases}$. Find the corresponding transfer function $\tilde{f}_{bell}(\omega)$.

D. Which of the above would you want to use?