This is a simple example of ICA applied to the cocktail party problem. We will generate a mixture of signals and then attempt to unmix them.

Q1. First, let’s generate the signals. We take two non-Gaussian sources, each of which can assume a value of -1 or 1 with a probability of 0.5. That is, a “typical” example of four samples is represented by the matrix

\[
S = \begin{pmatrix}
-1 & -1 \\
-1 & 1 \\
1 & -1 \\
1 & 1
\end{pmatrix}
\]

(Geometrically, these are positioned at the four corners of a square.) Our measured time series are represented by the columns of \( Y = SM \). To keep things really simple, we’ll assume that the mixing matrix \( M \) is the identity, \( M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \), so \( Y = S \).

Now, (forgetting that \( Y = S \)) we want to try to unmix \( Y \) to recover the sources \( S \). We first apply principal components to write \( Y = XA \), and then we will look for choices of \( A \) that maximize the non-Gaussian-ness of the columns \( X = YA^{-1} \)

That is, we look for the eigenvectors and eigenvalues of \( Y^T Y = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \). This is the identity, so any pair of orthonormal vectors can serve for \( A \). So we can write

\[
A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}
\]

Given this setup, what rotations (i.e., what values of \( \theta \)) maximize the kurtosis-squared of the columns of \( YA^{-1} \)? What rotations maximize the negentropy (minimize the entropy) of the columns of \( YA^{-1} \)?