Q1: Power spectra of some simple noises

A. Poisson noise. A Poisson noise \( n(t) \) is a sequence of delta-function pulses, each occurring independently, at some rate \( r \). (More formally, it is a sum of pulses of width \( \Delta \tau \) and height \( 1/\Delta \tau \), and the probability of a pulse between time \( t \) and \( t + \Delta t \) is \( r\Delta t \), and we consider the limit of \( \Delta \tau \to 0 \) and \( \Delta t \to 0 \).) Calculate the power spectrum \( P_n(\omega) \) of this noise.

B. Shot noise. A shot noise \( u(t) \) is a process in which copies of a stereotyped waveform \( x(t) \), occurring at random times, are superimposed. That is, \( u(t) = \sum_i x(t - t_i) \), where the times \( t_i \) are determined by a Poisson process of rate \( r \). The “shots” \( x(t) \) are typically considered to be causal, namely, \( x(t) = 0 \) for \( t < 0 \). Given the Fourier transform

\[
\tilde{u}(\omega) = \int_0^\infty u(t)e^{-i\omega t} dt,
\]

find the power spectrum \( P_u(\omega) \) of \( u \).

C. Shot noise, variable shot size. This is a process \( v(t) \) in which the amplitudes of the “shots” vary randomly. That is, \( v(t) = \sum_i a_i x(t - t_i) \), where the amplitudes \( a_i \) are chosen independently. Given the Fourier transform \( \tilde{v}(\omega) = \int_0^\infty u(t)e^{-i\omega t} dt \) and the moments of the distribution of the \( a_i \), find the power spectrum \( P_v(\omega) \) of \( v \).

Q2: Input and output noise

Recall the behavior of a linear system with additive noise (pages 16-17 of NAV notes), consisting of a linear filter \( G \) (characterized by its transfer function \( \tilde{g}(\omega) \):

\[
\begin{align*}
\text{s}(t) \quad &\xrightarrow{G} \quad \Sigma \quad &\xrightarrow{z(t)} \\
\text{r}(t) \quad &\xrightarrow{z(t)}
\end{align*}
\]
If the input is \( s(t) = \tilde{s}(\omega_0)e^{i\omega t} \) and there is an additive noise \( z(t) \) with power spectrum \( P_z(\omega) \), then the quantity \( \frac{1}{T} F(r, \omega_0, T, 0) = \frac{1}{T} \int_0^T r(t)e^{-i\omega t} dt \), when calculated for data lengths \( T \) that are a multiple of the period \( 2\pi / \omega_0 \), has a mean value \( \bar{s}(\omega_0)\tilde{g}(\omega_0) \) and a variance \( \frac{1}{T} P_z(\omega_0) \).

Analyze the situation when there is also some noise added prior to \( G \), diagrammed below: