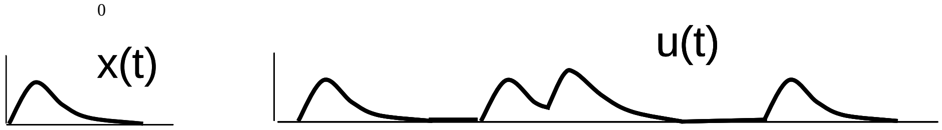
Noise and Variability

Homework #1 (2008)

Q1: Power spectra of some simple noises

A. Poisson noise. A Poisson noise $n(t)$ is a sequence of delta-function pulses, each occurring independently, at some rate r. (More formally, it is a sum of pulses of width $\Delta\tau$ and height $1/\Delta\tau$, and the probability of a pulse between time t and $t+\Delta t$ is $r\Delta t$, and we consider the limit of $\Delta\tau \to 0$ and $\Delta t \to 0$). Calculate the power spectrum $P_n(\omega)$ of this noise.

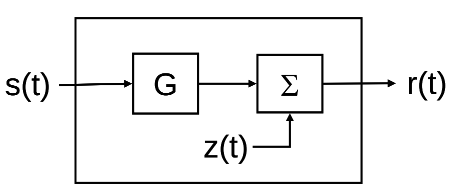
B. Shot noise. A shot noise $u(t)$ is a process in which copies of a stereotyped waveform $x(t)$, occurring at random times, are superimposed. That is, $u(t) = \sum_{t_i} x(t-t_i)$, where the times t_i are determined by a Poisson process of rate r. The "shots" $x(t)$ are typically considered to be causal, namely, $x(t)=0$ for $t<0$. Given the Fourier transform $\tilde{u}(\omega) = \int_0^\infty u(t)e^{-i\omega t}dt$, find the power spectrum $P_u(\omega)$ of u.



C. Shot noise, variable shot size. This is a process $v(t)$ in which the amplitudes of the "shots" vary randomly. That is, $v(t) = \sum_{t_i} a_i x(t-t_i)$, where the amplitudes a_i are chosen independently. Given the Fourier transform $\tilde{u}(\omega) = \int_0^\infty u(t)e^{-i\omega t}dt$ and the moments of the distribution of the a_i, find the power spectrum $P_v(\omega)$ of v.

Q2: Input and output noise

Recall the behavior of a linear system with additive noise (pages 16-17 of NAV notes), consisting of a linear filter G (characterized by its transfer function $\tilde{g}(\omega)$:



If the input is $s(t) = \tilde{s}(\omega_0)e^{i\omega_0 t}$ and there is an additive noise $z(t)$ with power spectrum $P_z(\omega)$, then the quantity $\frac{1}{T}F(r,\omega_0,T,0) \equiv \frac{1}{T}\int_0^T r(t)e^{-i\omega_0 t}dt$, when calculated for data lengths T that are a multiple of the period $2\pi/\omega_0$, has a mean value $\tilde{s}(\omega_0)\tilde{g}(\omega_0)$ and a variance $\frac{1}{T}P_z(\omega_0)$.

Analyze the situation when there is also some noise added prior to G, diagrammed below: