Linear Transformations and Group Representations

Homework #1 (2010-2011)

Q1: Eigenvectors of some linear operators in matrix form (also see Homework from "Algebraic Overview" (2008-2009))

In each case, find the eigenvalues, the eigenvectors, the dimensions of the eigenspaces, and whether a basis can be chosen from the eigenvectors. (1)

A. 
$$A = \begin{pmatrix} 1 & r \\ 0 & 1 \end{pmatrix}$$
.  
B.  $B = \begin{pmatrix} q & r \\ r & q \end{pmatrix}$  (assume  $q > r > 0$ ).

Q2: Eigenvectors of some linear operators in a continuous space

V is a vector space of functions of time. In each case, find the eigenvalues and eigenvectors of the indicated operator, and determine whether the operator is time-translation invariant

A. 
$$Lv(t) = tv(t)$$
.

B. 
$$Rv(t) = v(-t)$$
.

C. 
$$Mv(t) = \frac{d}{dt}v(t)$$
.

D. 
$$Nv(t) = t \frac{d}{dt}v(t)$$
.