Linear Systems, Black Boxes, and Beyond

Homework #2 (2012-2013)

Q1: Some standard windowing functions. For each of the windowing functions $W(t)$ below (each nonzero only on $[-L/2, L/2]$), calculate their Fourier transforms $\hat{W}(\omega) = \int_{-\infty}^{\infty} W(t) e^{-i\omega t} dt$ and characterize the asymptotic behavior of $\left|\hat{W}(\omega)\right|^2$ for large $|\omega|$. As mentioned in class, each windowing function represents a tradeoff between the sharpness of the peak of $\left|\hat{W}(\omega)\right|^2$ and the heaviness of its tails.

A. $W_{boxcar}(t) = \frac{1}{L}$ for $|t| \le L/2$, 0 otherwise

B. $W_{tent}(t) = \frac{2}{L}(1 - \frac{2|t|}{L})$ for $|t| \le L/2$, 0 otherwise

C. $W_{cosinebell}(t) = \frac{1}{L}\left(1 + \cos\left(\frac{2\pi t}{L}\right)\right)$ for $|t| \le L/2$, 0 otherwise