Linear Transformations and Group Representations

Homework #1 (2012-2013)

Q1: Eigenvectors of some linear operators in matrix form (also see Homework from "Algebraic Overview" (2008-2009))

In each case, find the eigenvalues, the eigenvectors, the dimensions of the eigenspaces, and whether a basis can be chosen from the eigenvectors.

A. $A=\begin{pmatrix} 1 & r \\ 0 & 1 \end{pmatrix}$.

B. $B=\begin{pmatrix} q & r \\ r & q \end{pmatrix}$ (assume $q>r>0$).

C. $C=\begin{pmatrix} q & r \\ -r & q \end{pmatrix}$

Q2: Eigenvectors of some linear operators in a continuous space

V is a vector space of functions of time. In each case, find the eigenvalues and eigenvectors of the indicated operator, and determine whether the operator is time-translation invariant

A. $Lv(t)=tv(t)$.

B. $Rv(t)=v(-t)$.

C. $Mv(t)=\frac{d}{dt}v(t)$.

Q3: Knowing vector lengths determines the inner product.

V is a Hilbert space, and $\langle v,w\rangle$ is its inner product. Write $\langle v,w\rangle$ in terms of the squared vector lengths $\|av+bw\|^2=\langle av+bw,av+bw\rangle$ for selected values of *a* and *b* Hint: consider especially $(a,b)=\{(1,1),\ (1,-1),\ (1,i),\ (1,-i)\}$.