Linear Systems, Black Boxes, and Beyond

Homework #4 (2014-2015), Questions

Q1: Covariances in a network

[Same set-up as Q1 of Homework 3] Given the following network, where F, G, and H are linear filters with transfer functions \( \tilde{F}(\omega) \), \( \tilde{G}(\omega) \), and \( \tilde{H}(\omega) \), and \( x(t) \) and \( y(t) \) are independent noise inputs with power spectra \( P_x(\omega) \) and \( P_y(\omega) \):

\[
\begin{align*}
F & \quad a(t) \\
+ & \quad b(t) \\
G & \quad c(t) \\
+ & \quad d(t) \\
H & \quad z(t)
\end{align*}
\]

A. Calculate the cross-spectra \( Z_{XP}(\omega) \) and \( Z_{YP}(\omega) \).

B. Now assume that \( x(t) \) and \( y(t) \) are NOT independent, and their dependence is characterized by a nonzero cross-spectrum \( X_{YP}(\omega) \). Calculate the power spectrum \( Z(\omega) \) in terms of \( X(\omega) \), \( P_y(\omega) \), and \( X_{YP}(\omega) \).

Q2. Multiple signals with common and private noise sources

Say there are \( N \) observed signals \( z_i(t) \), each of which is the result of adding a common noise source \( x(t) \), filtered by a linear filter \( F_i \), to a private noise source \( y_i(t) \), filtered by a linear filter \( G_i \). All the noises \( x(t) \) and \( y_i(t) \) are assumed independent.

A. Determine the cross-spectra \( P_{z_i z_j} \) in terms of the power spectra \( P_x \), \( P_{y_i} \), and the filter characteristics \( \tilde{F}_i \) and \( \tilde{G}_i \).

B. Now assume that all of the private noises \( y_i(t) \) are 0. Consider, for each frequency \( \omega \), the matrix \( P_{z_i z_j}(\omega) \). Does it have any special properties?