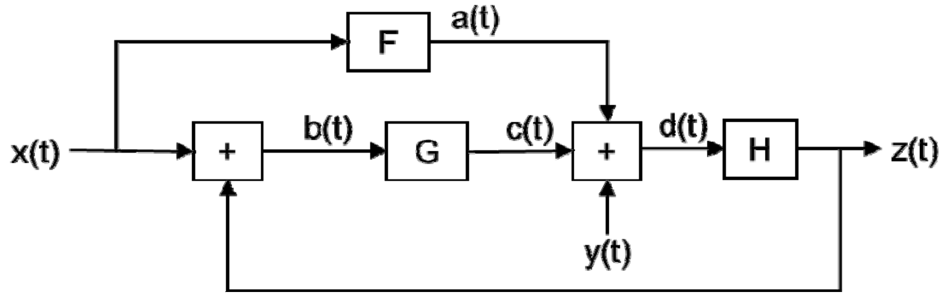
Linear Systems, Black Boxes, and Beyond

Homework #4 (2014-2015), Questions

Q1: Covariances in a network

[Same set-up as Q1 of Homework 3] Given the following network, where F, G, and H are linear filters with transfer functions $\tilde{F}(\omega)$, $\tilde{G}(\omega)$, and $\tilde{H}(\omega)$, and $x(t)$ and $y(t)$ are independent noise inputs with power spectra $P_X(\omega)$ and $P_Y(\omega)$:



A. Calculate the cross-spectra $P_{Z,X}(\omega)$ and $P_{Z,Y}(\omega)$.

B. Now assume that $x(t)$ and $y(t)$ are *NOT* independent, and their dependence is characterized by a nonzero cross-spectrum $P_{X,Y}(\omega)$. Calculate the power spectrum $P_Z(\omega)$ in terms of $P_X(\omega)$, $P_Y(\omega)$, and $P_{X,Y}(\omega)$.

Q2. Multiple signals with common and private noise sources

Say there are N observed signals $z_i(t)$, each of which is the result of adding a common noise source $x(t)$, filtered by a linear filter F_i, to a private noise source $y_i(t)$, filtered by a linear filter G_i. All the noises $x(t)$ and $y_i(t)$ are assumed independent.

A. Determine the cross-spectra P_{Z_i,Z_j} in terms of the power spectra P_X, P_{Y_i}, and the filter characteristics $\tilde{F}_i$ and $\tilde{G}_i$.

B. Now assume that all of the private noises $y_i(t)$ are 0. Consider , for each frequency ω, the matrix $P_{Z_i,Z_j}(\omega)$. Does it have any special properties?