Linear Transformations and Group Representations

Homework #1 (2014-2015), Questions

Q1: Eigenvectors of some linear operators (linear transformations) in matrix form

In each case, use the characteristic equation to find the eigenvalues, the eigenvectors, the dimensions of the eigenspaces, and whether a basis can be chosen from the eigenvectors.

A. $Q = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$, with $p \neq q$.

B. $A = \begin{pmatrix} 1 & r \\ 0 & 1 \end{pmatrix}$, with $r \neq 0$.

C. $B = \begin{pmatrix} q & r \\ r & q \end{pmatrix}$ (assume $q > r > 0$).

D. $C = \begin{pmatrix} q & r \\ -r & q \end{pmatrix}$

Q2: Eigenvectors of derived linear operators

Say X and Y are linear transformations from a vector space V to itself, and v is an eigenvector both of X, with eigenvalue λ_X , and of Y, with eigenvalue λ_Y .

A. Show that v is also an eigenvector of the transformation $X+Y$, and find its eigenvalue.

B. Show that v is also an eigenvector of the transformation αX ,where α is a scalar, and find its eigenvalue.

C. Show that v is also an eigenvector of the transformation XY , and find its eigenvalue.

D. Show that v is also an eigenvector of the transformation $XY - YX$, and find its eigenvalue.