Linear Transformations and Group Representations

Homework #3 (2014-2015), Questions

Representations of the dihedral group of D_4

Here we construct some of the representations of the dihedral group D_4, i.e., the rotations and reflections of a square. (These are not all of its representations, and these are not necessarily irreducible.) Let's Let's use the following notation for its elements:

I : the identity
R , R^{-1}: 90-degree rotations right and left ($R^4 = I$)
C : rotation by 180 deg ($C^2 = I$, $R^2 = C$)
X , Y : mirror flips in the *x*- and *y*-axes ($X^2 = Y^2 = 1$)
$M_\backslash$, $M_/$: mirror flips on the two diagonals ($M_\backslash^2 = M_/^2 = I$).

Here is the start of a character table, i.e., $\chi_L(g) = tr(L_g)$, where *g* is one of the $g \in \{I, R, R^{-1}, C, X, Y, M_\backslash, M_/\}$. Note that for group elements *g* and *h* that are intrinsically identical, i.e., related by $h = \alpha g \alpha^{-1}$, then $\chi_L(g) = \chi_L(h)$, so we've grouped them into a single column. For example, $R^{-1} = XRX^{-1}$, $Y = M_\backslash X M_\backslash^{-1}$, and $M_\backslash = XM_/X^{-1}$ (try it!).

When *L* is the trivial representation, L_g is the 1×1 identify matrix, whose trace is 1:

			g		
L	I	R , R^{-1}	C	X , Y	$M_\backslash$, $M_/$
Trivial rep.	1	1	1	1	1

A. Determine the characters for the representation based on 2×2 matrices that express the rotations and mirror-flips of the square in the standard coordinate plane.
B. Determine the characters based on permutation matrices, where we consider the four corners to be the objects permuted.
C. Determine the characters based on the permutation matrices, where we consider the four sides to be the objects permuted.
D. Determine the characters based on the permutation matrices, where we consider two diagonals to be the objects permuted.
E. Which of the above representations contain the trivial (identity) representation?