Multivariate Methods

Homework #1 (2014-2015), Answers

Q1: Regression and "cross-correlation analysis"
Consider the standard regression scenario described in the class notes, pages 1-2. That is, there are n observations, $y_1,\ldots,y_n$*, and* p *regressors, where the typical regressor* $\vec{x}_j$ *is a column* $x_{1,j},\ldots x_{n,j}$*, and the set of* p *regressors forms a* $n\times p$ *matrix* X *, and we seek a set of* p *coefficients* $b_1,\ldots,b_p$*, the* $p\times 1$ *matrix* B*, for which* $\left|Y-XB\right|^2$ *is minimized.*
Now let's assume that the regressors $\vec{x}_j$ *are orthonormal. For example, we're doing spatial receptive field analysis. Here* $x_{i,j}$ *corresponds to the luminance presented on the ith trial in pixel j, and we've designed our stimuli so that, over the entire stimulus sequence,* $\sum_{i=1}^{N} x_{i,j}x_{i,k}=0$ *if* $j\neq k$*, and* $\sum_{i=1}^{N} x_{i,j}x_{i,j}=1$.
How does this simplify the computation of the regressors B *?*

We have the formal solution $B=(X^*X)^{-1}X^*Y$. The assumptions of orthonormality, namely, $\sum_{i=1}^{N} x_{i,j}x_{i,k}=0$ if $j\neq k$, and $\sum_{i=1}^{N} x_{i,j}x_{i,j}=1$, mean that $(X^*X)_{j,k}=(X^TX)_{j,k}=\sum_i x_{i,j}x_{i,k}$. (Since the *x*'s are all real, $X^*=X^T$.) So X^*X is the identity matrix, and $B=(X^*X)^{-1}X^*Y=X^*Y$. That is, the model coefficients *B* can be computed by correlating the response sequence *Y* against the stimulus sequences *X*.

An extension of this argument leads to the "reverse correlation" procedure for determining the temporal aspects of receptive fields.