Multivariate Methods

Homework #2 (2014-2015), Questions

Q1: A three-stimulus brain in a two-stimulus world

Consider a toy functional imaging experiment, in which the brain has 3 pixels, and there are two stimuli. Say that stimulus 1 causes an activation of +2 units in pixel 1, and -1 unit in pixels 2 and 3; say that stimulus 2 causes an activation of +2 units in pixel 2, and -1 unit in pixels 1 and 3. So we have a 3×2 data matrix Y.

A. Compute its principal components, $Y = XB$, with the columns of X orthonormal, and the rows of B orthogonal (but not necessarily orthonormal).

Verify that $Y = XB$, where the columns of X orthonormal, and the rows of B orthogonal.

Q2. Rotation of principal components. Same setup as Q1. Let's see if we can find a simple way to unmix these components. Let $\vec{u}_1 = \vec{x}_1 \cos\theta + \vec{x}_2 \sin\theta$, $\vec{u}_2 = -\vec{x}_1 \sin\theta + \vec{x}_2 \cos\theta$. Since the $\vec{u}_i$ are a non-singular linear combination of the $\vec{x}_i$, they necessarily also account for the data matrix Y. We might consider a transformation to the $\vec{u}_i$ to be simpler if the coefficients in the $\vec{u}_i$ are smaller. The $\vec{u}_i$, like the $\vec{x}_i$, constitute the columns of a 3×2 matrix, $U(\theta)$. Is there a rotation θ that minimizes the sum of the squares of these 6 quantities? If so, find it; if not, explain why and suggest alternative strategies.