

Groups, Fields, and Vector Spaces

Homework #1 (2018-2019)

Q1: Group or not a group?

Which of the following are groups? If a group, is it commutative? Finite or infinite? If infinite, is it discrete or continuous? If not a group, where does it fail?

- A. The even integers $\{\dots -6, -4, -2, 0, 2, 4, 6, \dots\}$, under multiplication
- B. The set of all translations of 3-space, under composition
- C. The set of all rotations of 3-space, under composition
- D. The set of all $N \times N$ matrices with integer entries, under matrix addition
- E. The set of all $N \times N$ matrices with integer entries, under matrix multiplication
- F. The set of all 2×2 matrices with integer entries and determinant 1, under matrix multiplication
- G. Complex numbers, under addition
- H. Complex numbers, under multiplication

Q2. Modular arithmetic

For two integers x and y , we say $x = y \pmod{k}$ if x and y differ by an integer multiple of k . So, for example, $3+4=7 \pmod{5}$ and $6*9=54 \pmod{11}$.

- A. Show that the integers $\{0, 1, \dots, k-1\}$ form a group under addition \pmod{k} .
- B. For what integers k do the integers $\{1, \dots, k-1\}$ form a group under multiplication \pmod{k} ?