Linear Transformations and Group Representations

Homework #3 (2018-2019), Questions

Q1: Complete the character table of the group consisting of rotations and mirrorings of the cube.

rot90: rotation by 90 deg around any face
rot180: rotation by 180 deg around any face
edge flip: rotation by 180 deg around the line that connects the midpoints of two opposite edges
corner rot 120: rotation by 120 deg around the line that connects opposite corners
invert indicates that the above operation is followed by inversion through the center of the cube

Only the named representations are irreducible; the representations in red are NOT irreducible are starting points from which we remove a previously-determined irreducible representation by subtracting its character

element description	ident.	rot 90	rot 180	edge flip	corner rot 120	invert	rot 90 invert	rot 180 invert	edge flip invert	corner rot 120 invert	sum of squares
element count	1	6	3	6	8	1	6	3	6	8	
representation	**Character**										
trivial	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	48
	Below: representation based on parity of the number of inversions										
parity	+1	+1	+1	+1	+1	-1	-1	-1	-1	-1	48
	Below: representation based on permutations of the three opposite face pairs										
	+3	+1	+3	+1	0	+3	+1	+3	+1	0	96
	Below: remove the projection on the trivial										
face-pair	+2	0	+2	0	-1	+2	0	+2	0	-1	48
	Below: tensor product face-pair with parity										
face-pair ⊗ parity	+2	0	+2	0	-1	-2	0	-2	0	+1	48
	Below: representation based on 3D rotation matrices										
3D	+3	+1	-1	-1	0	-3	-1	+1	+1	0	48
	Below: tensor product 3D with parity										
3D ⊗ parity	+3	+1	-1	-1	0	+3	+1	-1	-1	0	48

Q2: Consider the same group as in Q1 as a permutation group on the 8 vertices. Compute the character of this representation, and determine if it is irreducible. If not, determine its decomposition into irreducible representations.