Q1: Graph Laplacians

Consider the following graph.

A. Write its graph Laplacian $L$.

B. Based on the symmetry of the graph, write a permutation matrix $P \neq I$ that commutes with $L$, for which $P^3 = I$.

C. Determine the eigenvalues and eigenvectors of $P$.

D. Using $PL = LP$, determine the eigenvectors and eigenvalues of $L$. 