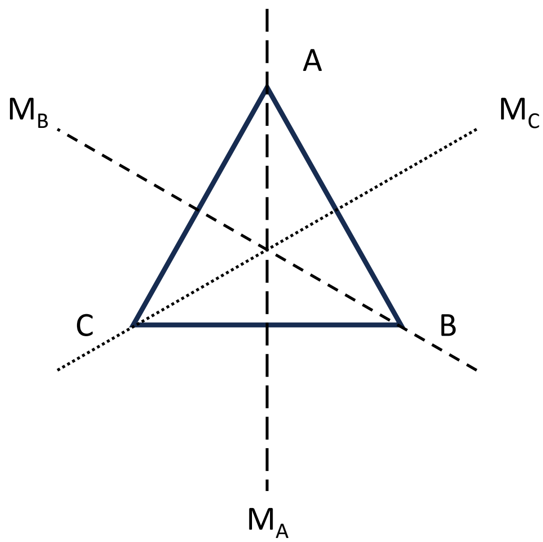
Groups, Fields, and Vector Spaces

Homework #1 (2024-2025), Questions

Q1: Multiple views of the same group: rotations and reflections of the triangle.



Consider the rotations and reflections of an equilateral triangle. Designate the identity transformation by *I*, a clockwise rotation of $\frac{2\pi}{3}$ by *R*, a counter-clockwise rotation of $\frac{2\pi}{3}$ by *S*, and the three mirror reflections (as diagrammed here) by M_A, M_B, and M_C.

i. Write out how these group elements act, viewing each group element *x* as the permutation $\pi(x)$ that maps a group element a to $x \circ a$.

Use standard permutation notation: Every permutation can be broken down into disjoint cycles, by following around repeated application of the permutation to one element. The permutation that maps E to F, F to G, and G to E is written (EFG) or, equivalently, (FGE) or GEF). The permutation that maps P to Q is written as (PS) or (QP). The combination of the two is written, for example, as (EFG)(PQ). An object Y that is mapped to itself may be omitted, or indicated as (Y).

ii. Write out how these group elements act, viewing each group element as acting on the vertices *A*, *B*, and *C*.

iii. Write out how these group elements act, viewing each of them as permuting the "front" and the "back" of the object.

iv. Consider these motions to be transformations of the plane, and write them out as 2×2 matrices.

v. Consider these motions to be transformations of an object in 3D, in which the "reflections" are half-circle rotations around one of the mirror lines in the diagram. Write them out as 3×3 matrices.

vi. Verify that the subset $T = \{I, R, S\}$ is a subgroup. Write out its left and right cosets. Are they the same?

vii. Verify that the subset $V_A = \{I, M_A\}$ is a subgroup. Write out its left and right cosets. Are they the same?