Homework #1 (2024-2025), Questions

Q1: The downward bias of entropy – near-worst-case scenario

Consider estimating the entropy of a binary variable, whose distribution is defined by p, where p is the probability of drawing a 0, and $1-p$ is the probability of drawing a 1. The true entropy is given by $H(p)=-p\log p-(1-p)\log(1-p)$. What is the expected value of the naïve ("plug-in") estimate of entropy, based estimating p from two samples? From 3 samples? Compare to $H(p)$.

Q2. Differential entropy of a multivariate Gaussian

Recall that the multivariate Gaussian distribution for a variable $\vec{x}$ (a column vector of length n) with mean zero and covariance matrix $\left\langle \vec{x} \bullet \vec{x}^T \right\rangle = V$ is given by $p_V(\vec{x})=\frac{1}{(2\pi)^{n/2}\sqrt{\det V}}\exp\left(-\frac{\vec{x}^T V^{-1}\vec{x}}{2}\right)$. What is the differential entropy $(\log_e)$ of $p_V(\vec{x})$?