Evaluation of Poor Performance and Asymmetry in the Farnsworth-Munsell 100-Hue Test

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A statistical method for the analysis of errors on the Farnsworth-Munsell 100-hue test is introduced. The extent of asymmetry of errors are summarized by two indices, I₁ and I₂, derived from Fourier analysis of the error scores for the individual caps. The second index, I₂, describes the (bipolar) color axis; the first index, I₁, describes (monopolar) asymmetry of performance. The present analysis differs from previous approaches based on Fourier analysis of the errors in two ways: (1) a procedure is introduced which corrects the indices I₁ and I₂ for the biases that result from the segmentation of the test into four boxes; (2) statistics for the significance of I₁ and I₂ are derived by a Monte Carlo procedure, which properly handles the complex interdependence of individual error scores for each cap. Invest Ophthalmol Vis Sci 29:476-481, 1988

The Farnsworth-Munsell 100-hue test (FM-100)¹ is a widespread and important means for the evaluation of color vision and its disorders. It is sufficiently sensitive to serve as a means to characterize both superior performance² and clinical defects, both of peripheral^{3,4} and central⁵ color processing.

The FM-100 test consists of four trays of colored caps. Each tray contains 21 or 22 colored caps, whose hues vary gradually throughout one quadrant of color space. The subject is asked to arrange each set of caps in order of hue. The test is graded by assigning an error score to each cap which expresses the degree of mismatch between the cap and its neighbors.

While normal performance on the FM-100 test is straightforward to define, quantitative characterization of abnormal performance is more elusive. A variety of automatic procedures for the analysis of defective performance have been proposed. These methods rest on Fourier analysis of the pattern of individual error scores for each cap as a means to quantitate asymmetry of performance.

However, the segmentation of the test into four boxes means that an uneven distribution of scores is expected from *random* performance;¹¹ this in turn introduces biases into axis determination. Further-

more, the complex interdependence of the individual error scores for each cap makes analysis of the statistical significance of the Fourier components difficult.

In this report, we refine the analysis of the FM-100 test in order to handle these difficulties. The main features of the analysis are: (1) correction of individual cap error scores for the biases introduced by the segmentation of the test into boxes; and (2) a rigorous computational approach to the evaluation of the significance of Fourier components derived from the individual cap error scores. We focus primarily on the evaluation of test results with a large number of errors; however, the method is not limited to this regime.

Materials and Methods

The raw error scores obtained from the FM-100 test constitute an irregular, cyclic sequence of integers. The information in the error scores is contained in their overall size and distribution, and not in the individual error score obtained for any single cap. Fourier analysis provides a natural means for summarizing this information:⁸ the overall level of errors is given by the zeroth Fourier component, and the sizes (amplitudes) and phases (directions) of the first few Fourier components describe the overall asymmetry. In particular, the second Fourier component describes the axis of asymmetries, which is key to distinguishing varieties of color vision disturbances.⁴

Definition of the Indices

The first step in scoring the FM-100 test is the determination of the error scores for each cap. As is

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customary,¹ the error score for a cap is two less than the sum of the absolute values of the differences between the cap number and the numbers of its two nearest neighbors. We will denote the error score for the jth cap by E_j. The sum of the errors for each cap (the total error score T) measures the overall level of performance:

$$T = \sum_{j=1}^{85} E_j$$
 (1)

The design of the FM-100 test is such that random performance will lead to quite different expected values of error scores at the various caps. ¹¹ To compensate for this, we introduce a relative error score for each cap, which is the quotient of the raw error score E_j and the error score expected from chance performance E_j^* . The relative error score R_j for each cap is given by

$$R_j = \frac{E_j}{E_i^*} \tag{2}$$

Use of the relative error score R_j compensates for the uneven distribution of error scores due to chance alone, which is a consequence of the finite length of each box. The calculation of expected error scores E_j^* under the assumption of random ordering is outlined in the Appendix.

The next step is to describe the asymmetries of the relative error scores R_j by Fourier analysis, as has previously been done for the analysis of the raw error scores E_j . 7,8,10 The kth Fourier component is denoted by a complex number A_k , whose amplitude indicates the amount of asymmetry and whose phase indicates the direction of the asymmetry. (The real and imaginary parts of A_k are the coefficients of the cosine- and sine-terms of the Fourier series.)

$$A_0 = \frac{1}{85} \sum_{j=1}^{85} R_j,$$

$$A_k = \frac{2}{85} \sum_{j=1}^{85} R_j e^{-2\pi i j k/85}, \quad k \neq 0$$
(3)

The zeroth Fourier component A_0 is the average relative error at each cap. The second Fourier component A_2 expresses the strength of the axis of asymmetry of the relative error scores. That is, a large A_2 indicates that caps in one portion of the color circle, as well as caps of opponent colors, are poorly ordered. A large A_1 indicates poor performance on one side of the color circle which does not correspond to similarly poor performance on opponent colors.

The Fourier components A_1 and A_2 describe the absolute amount of their respective kinds of asymmetries. For purposes of statistical comparisons, it is more convenient to focus on the amount of asymmetries.

metry relative to the total amount of error. For this purpose, we define the harmonic indices

$$I_k = \frac{A_k}{A_0} \tag{4}$$

The directions of the response asymmetries are specified by the phases of the complex numbers I_1 and I_2 ; the sizes of the asymmetries are determined by the absolute values $|I_1|$ and $|I_2|$.

Calculation of Expected Distribution of Harmonic Indices

To evaluate the statistical significance of an observed value of a harmonic index $|I_k|$, it is necessary to determine the probability that as large (or larger) value of the harmonic index would arise from a cap arrangement generated by errors randomly distributed throughout the 85 caps. In the limiting case in which performance is completely random, such arrangements may be simulated by randomizing the placement of caps into boxes (as was done by Kitahara and Kandatsu¹²).

However, the distribution of the amplitude of a harmonic index depends strongly on the total number of errors. A harmonic index which is significant in conjunction with a large total error may be insignificant in conjunction with a small total error (see for example Figure 3 of Benzschawel¹³). This is because errors *must* cluster when the total number of errors is small.

To quantitate the dependence of the distribution of harmonic indices on total error, we simulated cap arrangements that would result from an intermediate number of errors distributed uniformly (on the average) across the caps. Intermediate levels of error were simulated by assuming that nearest-neighbor swaps occurred at a particular frequency f per tray. Each successive nearest-neighbor swap operated on the arrangement of caps prior to the swap, and not on the numerical order of the caps. As average number of swaps per tray increased, the degree of disorder of the cap arrangements gradually approached that of completely random performance.

This procedure was followed for selected values of the swap-per-tray rate f, ranging from 1 to 1500. In each simulation, the exact number of swaps for a particular tray was determined from a Poisson distribution of mean f, to simulate the variability with which errors must occur, even given a constant error rate. (A Poisson distribution was used to model the assumption that each swap was an *independent* event.) For each value of f, 10,000 independent arrangements of the caps were constructed. From each such arrangement, the total error and the values of

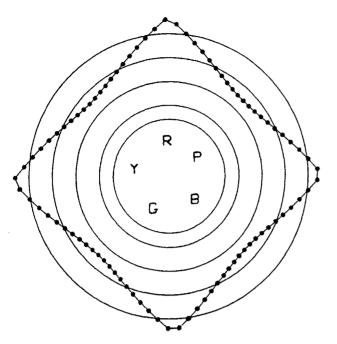


Fig. 1. Average random performance on the FM-100 test. The average error scores for each cap are calculated as described in the text; plotting is according to the original Farnsworth¹ method.

the harmonic indices were calculated, as described above.

Results

The Expected Error Score For Each Cap

The distribution of expected error scores E_j* anticipated under the assumption of completely random

performance is plotted in Figure 1. The distribution is highly nonuniform. The main deviation from non-uniformity is a near doubling of the expected error score near the ends of each tray. The differences in tray size (three trays of 21, one of 22) cause differences from quadrant to quadrant of less than 5%.

The Distribution of Total Error Scores

As the average number of swaps per tray f increases, the total error score increases from nearly perfect performance at f = 1 to nearly random performance at f = 1500 (Table 1). The calculated mean value of the total error score for an infinite number of swaps per tray (ie, completely random arrangements) agrees well with the exact value of 1202, obtained by summing the individual expected error scores E_i^* .

For all values of the average number of swaps per tray, the critical values for probabilities near 1 (high mean error) are displaced further from the median than are the critical values for corresponding probabilities near 0 (low mean error). That is, for any fixed level of performance f, the distribution of total error scores is skewed towards higher values. This is more prominent for values of f resulting in a mean total error score in the 50–300 range.

The distribution of total error scores of patient populations often demonstrates a skewness towards higher values.^{2,14} One possible contributor to this skewness is a positively skewed distribution of the underlying performance of individuals. However, the calculations summarized in Table 1 demonstrate that the FM-100 test itself introduces a positive skewness

Table 1. Distribution of total error for intermediate levels of performance

| Swaps/tray (f) | Mean total error | | CV | Critical values | | | | | | | | |
|-------------------|---------------------|------|-------|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | SD | | 0.001 | 0.010 | 0.025 | 0.050 | 0.500 | 0.950 | 0.975 | 0.990 | 0.999 |
| 1 | 15.2 | 7.4 | 0.491 | 0 | 0 | 4 | 4 | 16 | 29 | 33 | 37 | 45 |
| 2 | 29.3 | 10.1 | 0.346 | 4 | 8 | 12 | 12 | 28 | 49 | 53 | 57 | 65 |
| 4 | 54.0 | 13.3 | 0.247 | 20 | 24 | 28 | 32 | 52 | 77 | 81 | 89 | 101 |
| 6 | 75.7 | 15.6 | 0.207 | 32 | 40 | 48 | 52 | 76 | 105 | 109 | 117 | 133 |
| 10 | 112.2 | 19.3 | 0.172 | 56 | 68 | 76 | 80 | 112 | 145 | 153 | 161 | 177 |
| 20 | 181.5 | 26.2 | 0.145 | 108 | 124 | 132 | 140 | 180 | 225 | 237 | 245 | 269 |
| 40 | 277.0 | 35.8 | 0.129 | 176 | 196 | 208 | 220 | 276 | 337 | 353 | 369 | 397 |
| 60 | 348.2 | 43.0 | 0.124 | 228 | 256 | 268 | 280 | 348 | 425 | 437 | 457 | 493 |
| 100 | 454.3 | 53.5 | 0.118 | 304 | 340 | 356 | 372 | 452 | 549 | 569 | 589 | 645 |
| 200 | 633.2 | 68.7 | 0.109 | 432 | 484 | 504 | 524 | 632 | 753 | 773 | 805 | 869 |
| 300 | 749.7 | 79.6 | 0.106 | 512 | 576 | 600 | 624 | 748 | 832 | 913 | 945 | 1013 |
| 400 | 834.6 | 84.4 | 0.101 | 592 | 644 | 672 | 700 | 832 | 977 | 1005 | 1037 | 1093 |
| 600 | 952.4 | 91.3 | 0.096 | 696 | 740 | 776 | 804 | 952 | 1105 | 1133 | 1173 | 1237 |
| 1000 | 1085.2 | 93.1 | 0.086 | 792 | 872 | 904 | 936 | 1084 | 1241 | 1269 | 1305 | 1373 |
| 1500 | 1151.1 | 88.9 | 0.077 | 876 | 940 | 976 | 1004 | 1152 | 1297 | 1321 | 1353 | 1425 |
| inf. | 1204.2 | 84.7 | 0.070 | 948 | 1004 | 1036 | 1060 | 1208 | 1341 | 1369 | 1397 | 1465 |

Parameters of the distribution of total error T (equation (1)) for incompletely randomized arrangements of the Farnsworth-Munsell caps. Cap arrangements are generated by applying pairwise swaps of adjacent caps. The statistics for each average number of swaps per tray (f) are calculated from

10000 independent arrangements. The statistics for the last line of the table (infinite number of swaps per tray) were calculated from 10000 completely random cap arrangements. SD, standard deviation; CV, coefficient of variation (SD/mean).

Table 2. Critical values of harmonic indices error for intermediate levels of performance

| Mean total error | Swaps/tray (f) | Critical values | | | | | | | | |
|---------------------|-------------------|-----------------|----------|------------------|---------|-----------------|-------|-------|-------|--|
| | | | First ho | armonic | | Second harmonic | | | | |
| | | 0.500 | 0.950 | 0.990 | 0.999 | 0.500 | 0.950 | 0.990 | 0.999 | |
| | | | Cor | rected for finit | e boxes | | | | | |
| 15.2 | 1 | 0.450 | 0.997 | 0.997 | 0.997 | 0.439 | 0.987 | 0.987 | 0.987 | |
| 29.3 | 2 | 0.304 | 0.693 | 0.914 | 0.997 | 0.300 | 0.699 | 0.899 | 0.987 | |
| 54.0 | 4 | 0.213 | 0.464 | 0.590 | 0.749 | 0.208 | 0.460 | 0.583 | 0.742 | |
| 75.7 | 6 | 0.177 | 0.377 | 0.477 | 0.624 | 0.174 | 0.377 | 0.484 | 0.606 | |
| 112.2 | 10 | 0.148 | 0.311 | 0.391 | 0.484 | 0.143 | 0.303 | 0.385 | 0.485 | |
| 181.5 | 20 | 0.122 | 0.255 | 0.312 | 0.380 | 0.116 | 0.248 | 0.317 | 0.415 | |
| 277.0 | 40 | 0.108 | 0.224 | 0.270 | 0.327 | 0.098 | 0.215 | 0.268 | 0.342 | |
| 348.2 | 60 | 0.101 | 0.211 | 0.263 | 0.322 | 0.088 | 0.196 | 0.251 | 0.310 | |
| 454.3 | 100 | 0.092 | 0.194 | 0.239 | 0.305 | 0.078 | 0.176 | 0.226 | 0.281 | |
| 633.2 | 200 | 0.083 | 0.174 | 0.214 | 0.264 | 0.062 | 0.146 | 0.187 | 0.237 | |
| 749.7 | 300 | 0.076 | 0.159 | 0.198 | 0.238 | 0.055 | 0.130 | 0.170 | 0.225 | |
| 834.6 | 400 | 0.072 | 0.154 | 0.190 | 0.236 | 0.050 | 0.120 | 0.156 | 0.198 | |
| 952.4 | 600 | 0.065 | 0.136 | 0.172 | 0.207 | 0.044 | 0.109 | 0.144 | 0.183 | |
| 1085.2 | 1000 | 0.057 | 0.121 | 0.151 | 0.187 | 0.042 | 0.098 | 0.128 | 0.162 | |
| 1151.1 | 1500 | 0.052 | 0.112 | 0.141 | 0.171 | 0.042 | 0.094 | 0.121 | 0.157 | |
| 1204.2 | inf. | 0.032 | 0.101 | 0.128 | 0.166 | 0.042 | 0.097 | 0.124 | 0.152 | |
| 1204.2 | 1111. | 0.040 | | orrected for fin | | 0.015 | 0.077 | 0.121 | 0.132 | |
| | | 0.430 | | | | 0.425 | 0.007 | 0.007 | 0.007 | |
| 15.2 | 1 | 0.439 | 0.997 | 0.997 | 0.997 | 0.435 | 0.987 | 0.987 | 0.987 | |
| 29.3 | 2 | 0.299 | 0.685 | 0.913 | 0.997 | 0.298 | 0.680 | 0.895 | 0.987 | |
| 54.0 | 4 | 0.208 | 0.455 | 0.586 | 0.740 | 0.207 | 0.450 | 0.566 | 0.708 | |
| 75.7 | 6 | 0.175 | 0.374 | 0.468 | 0.619 | 0.174 | 0.367 | 0.471 | 0.590 | |
| 112.2 | 10 | 0.144 | 0.304 | 0.385 | 0.469 | 0.143 | 0.297 | 0.366 | 0.460 | |
| 181.5 | 20 | 0.120 | 0.249 | 0.308 | 0.370 | 0.116 | 0.241 | 0.300 | 0.377 | |
| 277.0 | 40 | 0.105 | 0.220 | 0.268 | 0.329 | 0.099 | 0.207 | 0.257 | 0.313 | |
| 348.2 | 60 | 0.099 | 0.208 | 0.257 | 0.306 | 0.090 | 0.190 | 0.233 | 0.282 | |
| 454.3 | 100 | 0.093 | 0.192 | 0.238 | 0.298 | 0.080 | 0.171 | 0.212 | 0.265 | |
| 633.2 | 200 | 0.084 | 0.176 | 0.215 | 0.260 | 0.064 | 0.140 | 0.178 | 0.220 | |
| 749.7 | 300 | 0.078 | 0.164 | 0.201 | 0.243 | 0.057 | 0.125 | 0.157 | 0.207 | |
| 834.6 | 400 | 0.075 | 0.157 | 0.195 | 0.242 | 0.052 | 0.115 | 0.147 | 0.182 | |
| 952.4 | 600 | 0.068 | 0.143 | 0.178 | 0.218 | 0.045 | 0.104 | 0.134 | 0.170 | |
| 1085.2 | 1000 | 0.059 | 0.126 | 0.155 | 0.192 | 0.041 | 0.093 | 0.118 | 0.151 | |
| 1151.1 | 1500 | 0.054 | 0.115 | 0.147 | 0.177 | 0.041 | 0.090 | 0.115 | 0.145 | |
| 1204.2 | inf. | 0.049 | 0.105 | 0.132 | 0.164 | 0.043 | 0.093 | 0.116 | 0.143 | |

Critical values of the distribution of the amplitude of the first and second harmonic indices $|I_1|$ and $|I_2|$ (equation (4)). The statistics for each degree of randomization are calculated from 10,000 independent arrangements, as in

Table 1. Harmonic indices are measured with or without the correction (2) for finite box sizes.

to the total error scores, even when the underlying performance is normally distributed.

Distribution of Harmonic Indices

The critical values of the distribution of absolute values of harmonic indices $|I_1|$ and $|I_2|$ are shown in Table 2. The distribution of the two indices are nearly identical. As expected from the lack of independence of error scores when the error rate is low, the critical values for harmonic indices are substantially higher when the total error score is low. For example, consider the 5% significance level (critical value of 0.95) for $|I_1|$. This is approximately 0.1 for fully random performance, but is 0.2 when the total error score is 400, and 0.3 when the total error score is 100.

Since common practice has been to calculate the harmonic indices based on raw error scores, the critical values of $|I_1|$ and $|I_2|$ were also calculated without the correction for the nonuniform distribution of

random performance (equation (2)). These results are presented in the second half of Table 2. The correction (2) makes very little difference in the critical values of $|I_1|$ and $|I_2|$. This is because the indices are ratios of raw Fourier components to the total error score, and not the raw Fourier components themselves. (However, since the distribution of expected error scores has an approximate periodicity of four around the color circle, the correction has a very large effect on the amplitude of the fourth index $|I_4|$).

The correction (2) is rigorously justified only in the limit of a total error score approaching 1200. It remains appropriate when the caps are arranged according to a nonrandom but systematic scheme (for example, scotopic brightness), provided that deviation from Munsell hue order produces a high total error score. For lower total error scores, the rigorously appropriate correction is the expected error score at each cap for that particular level of performance. This

correction would be intermediate between the correction (2) and no correction at all. But the similarity in the critical values of $|I_1|$ and $|I_2|$ calculated with and without correction (Table 2) shows that for most purposes, this secondary refinement is unnecessary.

Discussion

The present approach uses computer simulation to handle some of the statistical complexities that stem from the design of the FM-100. By simulating performance with errors of various frequencies distributed uniformly across the caps, we obtained statistics for the distribution of total error scores (Table 1) and of harmonic indices (Table 2). The statistics of Table 1 define the variability of test scores given a fixed level of underlying error; this table can be used to determine whether the overall level of performance in two sets of test results is significantly different. The statistics of Table 2 define the distribution of the sizes of the harmonic indices $|I_1|$ and $|I_2|$, under the hypothesis that error is uniformly distributed across the caps. The critical values of these distributions allow testing the hypothesis that an observed harmonic index represents random fluctuation without an underlying asymmetry of performance. In particular, a significant color axis must have a value of |I₂| which exceeds the P = 0.95—critical value for the measured level of total error.

The Correction For Finite Box Size

The present approach for the determination of color axis shares a reliance on Fourier analysis with the approaches previously proposed by other authors. 7,8,10 The refinement proposed here is that Fourier coefficients are calculated for the relative error R_j , rather than the actual error score E_j . The relative error R_j is related to the actual error score E_j by division by the expected error score at each cap E_j^* .

Because the caps are grouped into finite boxes, the expected error scores E_j^* are not uniform. The major effect of this correction is on the direction of the indices I_1 and I_2 , and not on their amplitude. Typically, the correction amounts to 0.5 to 1.0 caps. The effect of this correction is small if the error pattern is a sharp, narrow peak; it is larger if the error peak is wider, and particularly if each pole of the error distribution has a bimodal "swallowtail" shape. Although small, these corrections are comparable to the narrow separation of the extremes of the axis distributions of protans and deutans. ¹⁴ The correction always shifts the measured color axis towards the center of the box that contains it.

An Illustration

We apply the present technique to the protan and deutan data in the Farnsworth¹ manual. For the protan, the total error score is 136. The amplitude of the first harmonic index $|I_1|$ is 0.09. As seen in Table 2, this is well within the range expected by chance alone for this level of total error (P = 0.95, critical value)approximately 0.28). The amplitude of the second harmonic index $|I_2|$ is 0.54, which is highly significant; it exceeds the P = 0.999 critical value of approximately 0.43. The second harmonic axis points to cap 21.1. Without the correction (2) for finite box size, $|I_1|$ is also insignificant (0.07), $|I_2|$ remains highly significant (0.60), and the second harmonic axis is cap 21.0. For the deutan, the total error score is 174, $|I_1|$ is 0.09, $|I_2|$ is 0.74, and the axis is 15.0. Without the correction for finite box size, $|I_1|$ is 0.08, $|I_2|$ is 0.73, and the axis is 15.9. For both protan and deutan, the calculated axes agree closely with those given in the Farnsworth¹ manual, and the monopolar asymmetry I₁ is seen to be insignificant.

Ease of Application

Our calculations have used the original method of Farnsworth¹ for basic scoring. However, for practical purposes, the statistics are equally applicable to scores obtained by the method of Kinnear,¹⁵ because these two scoring methods give virtually identical values for harmonic indices.¹⁶ The calculation of I_1 and I_2 are easily integrated into existing computer analyses of the FM-100 test^{13,17,18} by inclusion of equation (2). However, since the amplitudes $|I_1|$ and $|I_2|$ are influenced only slightly by inclusion of (2), this correction is unnecessary for the purpose of assessing significance: the critical values of Table 2 may be applied to the uncorrected harmonic indices.

FORTRAN source code which includes all the calculations described in the text, as well as more extensive statistical tables, are available from the author.

Key words: color vision, Farnsworth-Munsell test, Fourier analysis

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Appendix

Here we calculate the average error score E* for the jth cap, under the assumption of completely random performance.

Consider a cap numbered j, sitting in a tray of N_{tray} caps inclusively numbered from N_{lo} to N_{hi} . The situations in which the cap is placed on the interior of a tray versus on

the end of a tray must be considered separately. If cap j is placed in the interior of its tray, its two neighbors will be drawn with equal probability from the set S of remaining caps: $\{N_{lo}, \ldots, j-1, j+1, \ldots, N_{hi}\}$. In this case, the average error score will be the expected value of the sum of the differences of the cap number (j) and those of its two neighbors. This quantity is the expected value of the sum of two random selections (without replacement) from the set $\{|N_{lo}-j|, \ldots, |-1|, |+1|, \ldots, |N_{hi}-j|\}$, ie, twice the average of this set.

If cap j is placed at the beginning of its tray, one of its two neighbors will be drawn with equal probability from the set S (above) of remaining caps in the tray. The other cap will be drawn with equal probability from the caps in the previous tray, which contains caps N'_{lo} to N'_{hi} . In this case, the sum of the differences of the cap number (j) and those of its two neighbors will be the sum of two components. One component is the difference of the cap number and that of its successor (within the tray); this component is the average of the numbers $\{|N_{lo}-j|,\ldots,|-1|,|+1|,\ldots,|N_{hi}-j|\}$. The second component is the difference of the cap number and that of its predecessor (from the previous tray); this component is the average of the numbers $\{|N'_{lo}-j|,\ldots,|N'_{hi}-j|\}$. A similar calculation holds for the case in which the cap j is placed at the end of its tray.

In a random arrangement, the probability that any particular cap will be placed on the interior of its tray is $(N_{tray} - 2)/N_{tray}$, and the probability that the cap will be placed at each end is $1/N_{tray}$. The overall expected sum-of-differences score for each cap is the sum of the expected sum-of-differences for each case considered above, weighted by the probability with which each case occurs. The expected error score E_j^* is two less than this weighted sum.

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