LOW- AND HIGH-ORDER NATURAL IMAGE STATISTICS VIEWED THROUGH TWO-DIMENSIONAL HERMITE FUNCTION FILTERS

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Introduction

The statistics of natural images constrain the organization of the visual system. Second-order statistics of natural images are well described by a power spectrum of 1/f, consistent with distance-independent scaling (Field, 1987). The center-surround organization of receptive fields characteristic of neurons in the early visual pathways can be viewed as a means to remove the redundancy associated with this low-frequency bias. Higher-order spectra show that natural images have statistical structure beyond second-order, but it is difficult to analyze these statistics because of their high dimensionality.

Two-dimensional Hermite functions (TDHs) have mathematical properties that recommend them for analysis of image statistics (Victor and Knight 2003). At each spatial scale, they form a complete basis set of steerable filters that are equally localized in space and spatial frequency (and in this sense, halfway between points and gratings). As shown below, TDHs can be organized into a set of orthogonal ranks of increasing spatial complexity. The lowest-rank (simplest) TDH is a Gaussian; higher-rank TDH functions include not only Gabor-like patches, but also patches with multiple orientations.

Methods

Step 1: We randomly chose 252 natural images from van Hateren (1998) database, and divided them into 3 groups with 84 images each. Each image is 1024 by 1024 pixels with 16-bit grayscale.

Step 2: We convolved each image with polar TDH filters up to rank 7, with 3 sizes of scale. Image statistics were calculated at 250-15,000 sample positions from each image.

Second-order image statistics across ranks and scales

Each rank consists of not only Cartesian and polar filters, but also intermediate filters. The full set of unit-magnitude filters at rank r corresponds to the surface of a sphere in (r+1)-space. Statistics for all of these filters are readily calculated via the steerable property.

Step 3: We used the steerable property of TDH filters to calculate convolutions of the images with the Cartesian TDH functions from linear combinations of the convolution of the images with polar TDH functions. The coefficients (p, q, r, s) in the linear combination are coefficients in the linear combination.

Results

Higher-order image statistics at rank 2

At rank 3, skewness and kurtosis also showed simple patterns of dependence on direction in filter space, with largest skewness for the filter functions sensitive to vertical gradients, and relatively uniform distribution of kurtosis.

Summary and Conclusion

- As expected from previous studies, second-order statistics are consistent with an isotropic, 1/f power spectrum, across six octaves.
- Higher-order statistics, as captured by the skewness and kurtosis of TDH filter functions, show approximate independence of spatial scale.
- Skewness is large only for even-rank filter functions whose mean is nonzero, and is similar across all such Cartesian and polar functions, across ranks. Conversely, kurtosis is large for the zero-mean filters, and small for the filters whose mean is nonzero.
- Skewness is small but positive for Cartesian odd-rank functions with a horizontal axis and for the unique odd-rank polar functions with a single inversion axis along the horizontal (*). These functions are sensitive to top-to-bottom luminance gradients.
- Third- and fourth-order statistics thus reveal novel, specific patterns of scale invariance.

References


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