LOW- AND HIGH-ORDER NATURAL IMAGE STATISTICS VIEWED THROUGH TWO-DIMENSIONAL HERMITE FUNCTION FILTERS

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Introduction

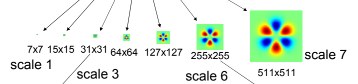
The statistics of natural images constrain the organization of the visual system. Second-order statistics of natural images are well-described by a power spectrum of $1/f^2$, consistent with distance-independent scaling (Field, 1987). The center-surround organization of receptive fields characteristic of neurons in the early visual pathways can be viewed as a means to remove the redundancy associated with this low-frequency bias. Higher-order spectra show that natural images have statistical structure beyond second-order, but it is difficult to analyze these statistics because of their high dimensionality.

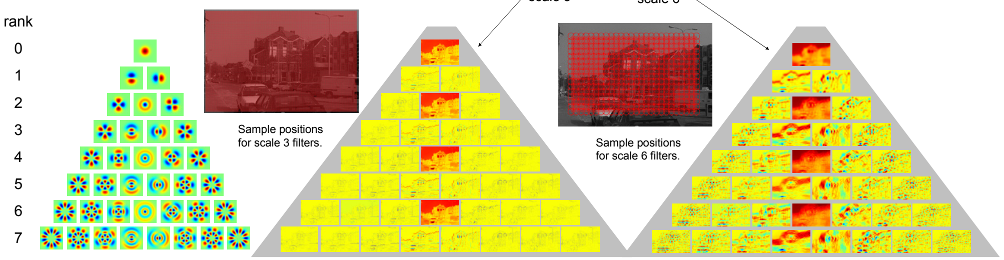
Two-dimensional Hermite functions (TDH's) have mathematical properties that recommend them for analysis of image statistics (Victor and Knight 2003). At each spatial scale, they form a complete basis set of steerable filters that are equally localized in space and spatial frequency (and in this sense, halfway between points and gratings). As shown below, TDH's can be organized into a set of orthogonal ranks of increasing spatial complexity. The lowest-rank (simplest) TDH is a Gaussian; higher-rank TDH functions include not only Gabor-like patches, but also patches with multiple orientations.

Methods

Step 1. We randomly chose 252 natural images from van Hateren (1998) database, and divided them into 3 groups with 84 images each. Each image is 1536 by 1024 pixels with 16-bit grayscale.

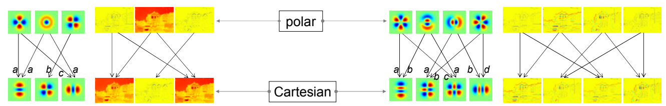
Step 2. We convolved each image with polar TDH filters up to rank 7 (36 functions), across 6 octaves of scale. Image statistics were calculated at 200-15,000 sample positions from each image.





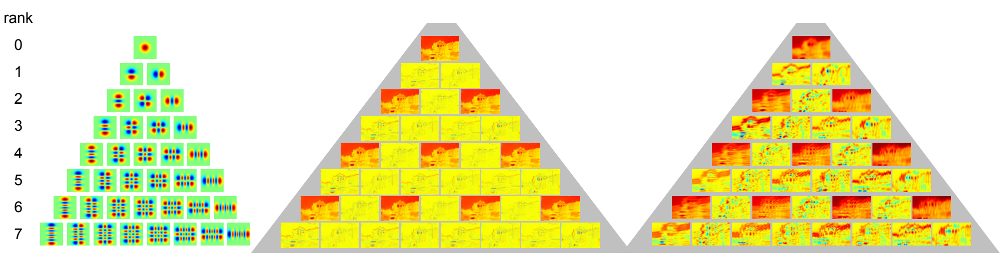
polar TDH filters | convolutions with polar TDH filters, scale 3 | convolutions with polar TDH filters, scale 6

Step 3. We used the steerable property of TDH filters to calculate convolutions of the images with the Cartesian TDH functions from linear combination of the convolution of the images with polar TDH functions.



$a=\sqrt{2}/2$, $b=1$, $c=-\sqrt{2}/2$ are coefficients in the linear combination.

$a=1/2$, $b=\sqrt{3}/2$, $c=-\sqrt{3}/2$, $d=-1/2$ are coefficients in the linear combination.



Cartesian TDH filters | convolutions with Cartesian TDH filters, scale 3 | convolutions with Cartesian TDH filters, scale 6

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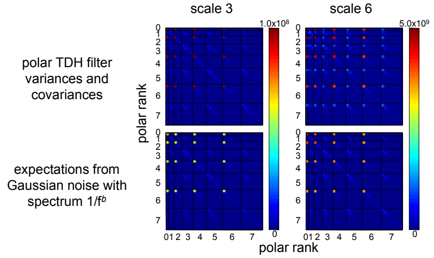
Results

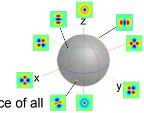
Second-order image statistics across ranks and scales



Second-order statistics were consistent with expectations of a $1/f^2$ power spectrum (Field, 1987) across ranks and scales. Best fit was obtained with a power of b=1.992.

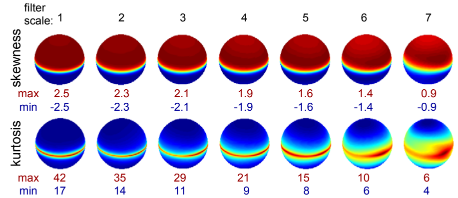
Higher-order image statistics at rank 2

Each rank consists of not only Cartesian and polar filters, but also intermediate filters. The full set of unit-magnitude filters at rank r corresponds to the surface of a sphere in (r+1)-space. Statistics for all of these filters are readily calculated via the steerability property.



The space of all unit-magnitude rank 2 filters

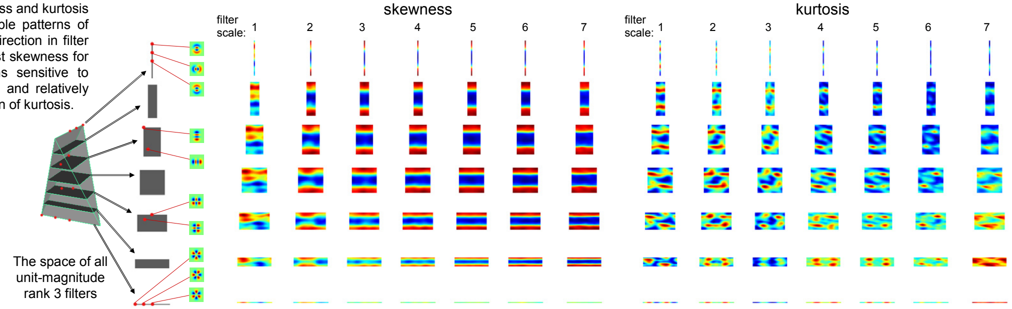
To examine higher-order statistics, we calculated the skewness and kurtosis of the filter outputs, as a function of rank and direction in filter space. At rank 2, skewness was independent of orientation, and largest for the circularly-symmetric (target-like) filters, while kurtosis was smallest for the target-like filters (at the poles of the diagram), and largest for filters on the equator of the diagram.



filter scale:	1	2	3	4	5	6	7
skewness max	2.5	2.3	2.1	1.9	1.6	1.4	0.9
skewness min	-2.5	-2.3	-2.1	-1.9	-1.6	-1.4	-0.9
kurtosis max	42	35	29	21	15	10	6
kurtosis min	17	14	11	9	8	6	4

Higher-order image statistics at rank 3

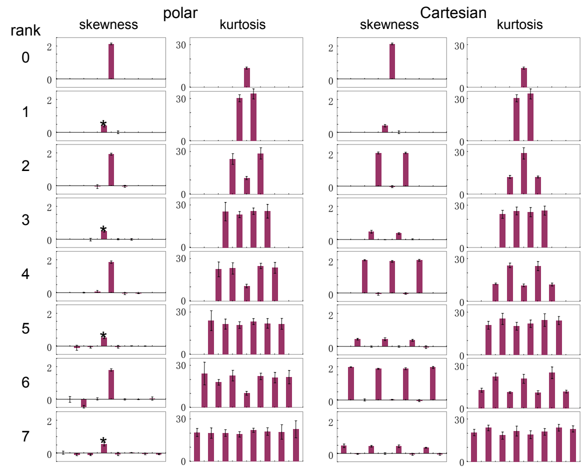
At rank 3, skewness and kurtosis also showed simple patterns of dependence on direction in filter space, with largest skewness for the filter functions sensitive to vertical gradients, and relatively uniform distribution of kurtosis.



The space of all unit-magnitude rank 3 filters

	skewness							kurtosis						
filter scale:	1	2	3	4	5	6	7	1	2	3	4	5	6	7
max	0.32	0.47	0.47	0.66	0.73	0.76	0.58	47	33	28	21	14	10	6
min	-0.32	-0.47	-0.47	-0.66	-0.73	-0.76	-0.58	35	25	19	15	10	7	4

Skewness and kurtosis in cardinal directions



The above analysis is at scale 3. Similar results were found at other scales. Error bars represent 1 s.d. across the three groups of 84 images.

Summary and Conclusion

- As expected from previous studies, second-order statistics are consistent with an isotropic $1/f^2$ power spectrum, across six octaves.
- Higher-order statistics, as captured by the skewness and kurtosis of TDH filter functions, show approximate independence of spatial scale.
- Skewness is large only for even-rank filter functions whose mean is nonzero, and is similar across all such Cartesian and polar functions, across ranks. Conversely, kurtosis is large for the zero-mean filters, and small for the filters whose mean is nonzero.
- Skewness is small but positive for Cartesian odd-rank functions with a horizontal inversion axis, and for the unique odd-rank polar functions with a single inversion axis along the horizontal (*). These functions are sensitive to top-to-bottom luminance gradients.
- Third- and fourth-order statistics thus reveal novel, specific patterns of scale invariance.
- These findings suggest that analysis via TDH functions provides a concise description of higher-order statistics of natural images.

References

Field, D.J. (1987). Relations between the statistics of natural images and the response properties of cortical cells. J Opt Soc Am [A], 4 (12), 2379-2394.

Victor, J.D., & Knight, B.W. (2003). Simultaneously band and space limited functions in two dimensions, and receptive fields of visual neurons. In: E.Kaplan, J. Marsden, & K.R. Sreenivasan (Eds.), Springer Applied Mathematical Sciences Series (pp. 375-420): Springer.

van Hateren, J.H., van der Schaaf, A. (1998) Independent component filters of natural images compared with simple cells in primary visual cortex. Proc.R.Soc.Lond. B 265:359-366.