IMPLICATIONS FOR THE DYNAMICS OF TEMPORAL LOBE SEIZURES DERIVED FROM MULTIVARIATE AUTOREGRESSIVE ANALYSIS OF ELECTROENCEPHALOGRAMS.

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INTRODUCTION

- We present a novel method of spatiotemporal signal analysis that exploits the dynamical relationships among underlying signal generators
- A hierarchy of temporally dependent generators is obtained from the spatiotemporal signal
- The method is applied to the analysis of human ictal EEG data and the derived "neural generators" are characterized by their non-linear dynamics

RATIONALE

The motivation for our analysis of spatiotemporal data is to remedy a shortcoming in principal components analysis (PCA), namely the non-uniqueness of the extracted components. We model the dynamic relationships between principal components using a multivariate extension of the standard linear autoregression (MLAR), and apply a novel hierarchical decomposition (HD) to address the drawback imposed by PCA. A detailed discussion of this approach follows.

PRINCIPAL COMPONENTS ANALYSIS

Principal components analysis (PCA) is a general approach to represent a spatiotemporal signal as orthogonal components. The components are ordered by the amount of signal variance they represent, thereby reducing a high-dimensional signal to only a few meaningful dimensions that can explain most of the variance. However, in the process of resolving orthogonal components, PCA ignores any dynamical relationships between the underlying signal generators.

- An M-channel spatiotemporal signal with N time points, X (M x N), is approximated by a second spatiotemporal signal Y (M x N), where Y consists of linear combinations of P orthogonal "principal components."
  - $Y = C^{\mathrm{T}}WT$
    - C (P x M) are spatial weights, T (P x N) are temporal weights (components), and W (P x P) is a diagonal matrix (the eigenvalues) whose elements, squared, are the amount of variance explained by each principal component (PC).
  - $R_{\mathrm{PCA}} = \mathrm{tr}[(X-Y)(X-Y)^{\mathrm{T}}]$ (the unexplained variance between X and Y) is minimized

MULTIVARIATE LINEAR AUTOREGRESSION

As noted, the dynamic relations among the underlying generators are overlooked by PCA. In order to make use of these relationships, we created multivariate linear autoregressive (MLAR) models of the first P components.

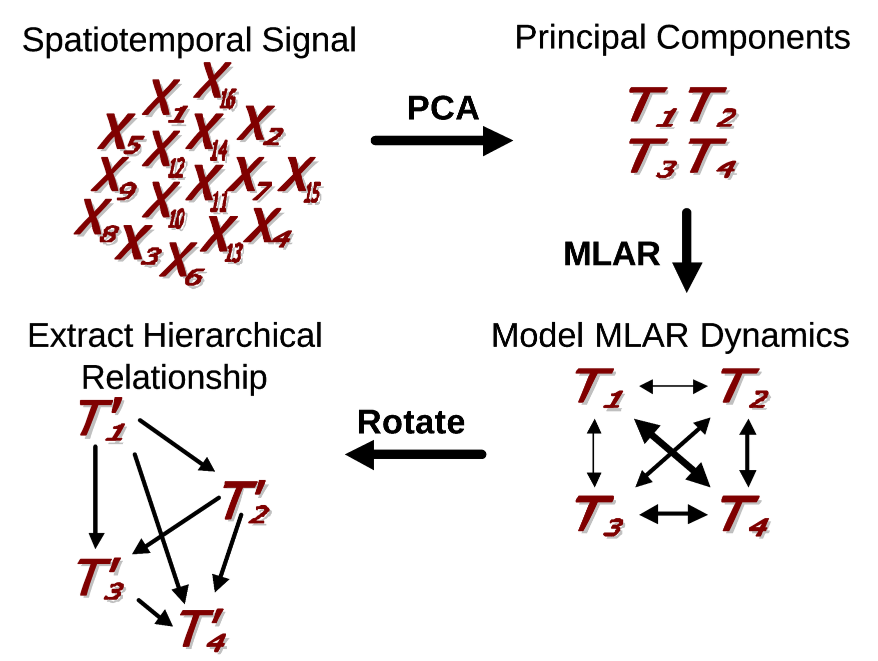
- The resolved temporal components, T, are represented by an MLAR model obtained through minimization of the square of the residual values in the model, R_{MLAR}^2
  - $T_{p,n} = R_{p,n} - m_p - \sum_{q=1}^{P}\sum_{l=1}^{s} A_{p:q,l}\, T_{q,n-l}$
    - Each one of s matrices A_l contains regression coefficients that describe the linear relationship between a component p and all components q at l time lags in the past. s is statistically determined by the Akaike criterion (AIC; Akaike 1974).

HIERARCHICAL DECOMPOSITION

Here, we introduce the hypothesis that the observed spatiotemporal data set reflects a hierarchically dependent set of underlying generators. Since the set of all regression coefficients, A, characterizes the signal dynamics, we sought rotations of the PCs that were consistent with a hierarchical interrelationship of the autoregressive coefficients across components. The hierarchical relationship among generators corresponds to a triangular form for all matrices A.

- Search for the rotation M that simultaneously triangularizes, as best as possible, all s regression matrices A (P x P), and apply the result to the PCs.
  - $A'_{\mathrm{tri}} = M^{\mathrm{T}}AM$
  - $T'_{\mathrm{rotated}} = M^{\mathrm{T}}T$

RATIONALE



HIERARCHICAL DECOMPOSITION

The HD method obtains the rotation that most nearly triangularizes all matrices A – and thereby transforms the components T into hierarchical form – through application of sequential, modified Jacobi rotations, J. The standard Jacobi rotation is a plane rotation about a pair of axes i,j designed to zero the element $A_{i,j}$; the modified Jacobi rotation applied along axes i,j minimizes the sum of squared elements below the diagonal (the residuals for the HD, R_{HD}).

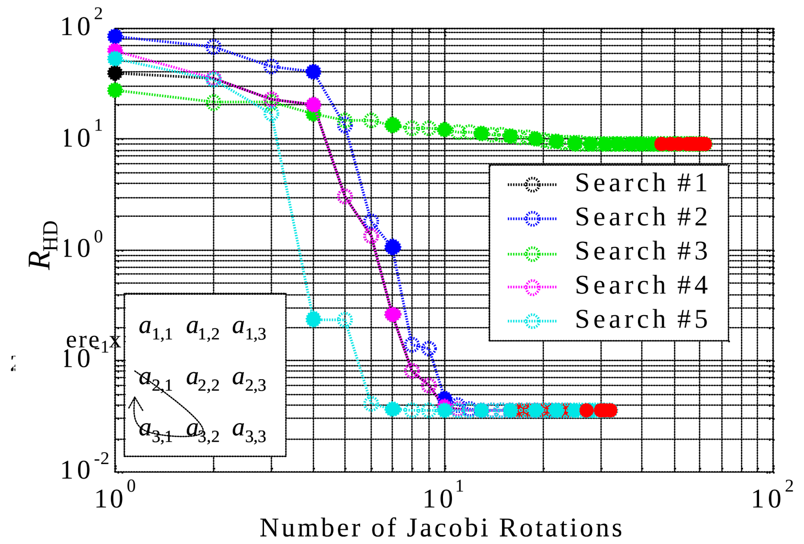
- $$J_{4,2}(q) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(q) & 0 & \sin(q) \\ 0 & 0 & 1 & 0 \\ 0 & -\sin(q) & 0 & \cos(q) \end{bmatrix}$$
- $A'_{\mathrm{tri}} = J_{i,j}(\theta_k)^{\mathrm{T}} \ldots J_{3,2}(\theta_2)^{\mathrm{T}} J_{2,1}(\theta_1)^{\mathrm{T}} A J_{2,1}(\theta_1) J_{3,2}(\theta_2) \ldots J_{i,j}(\theta_k)$
- $M = J_{2,1}(\theta_1) J_{3,2}(\theta_2) \ldots J_{i,j}(\theta_k)$
- $R_{\mathrm{HD}} = \sum_{l=1}^{s}\sum_{j=1}^{i-1}\sum_{i=2}^{P} A'_{l,i,j}$

A rotation exists which achieves $R_{\mathrm{HD}} = 0$ for a single matrix only when the eigenvalues of the matrix are all real (Mirsky 1990). In general this rotation will not simultaneously achieve $R_{\mathrm{HD}} = 0$ for a second matrix. Thus, the rotation M that "triangularizes" A is that which minimizes R_{HD}.

The minimization of R_{HD} is analogous to a multi-dimensional gradient descent. Since it is possible to get stuck in a local minimum, it is necessary to begin the search from numerous places in the rotation space.

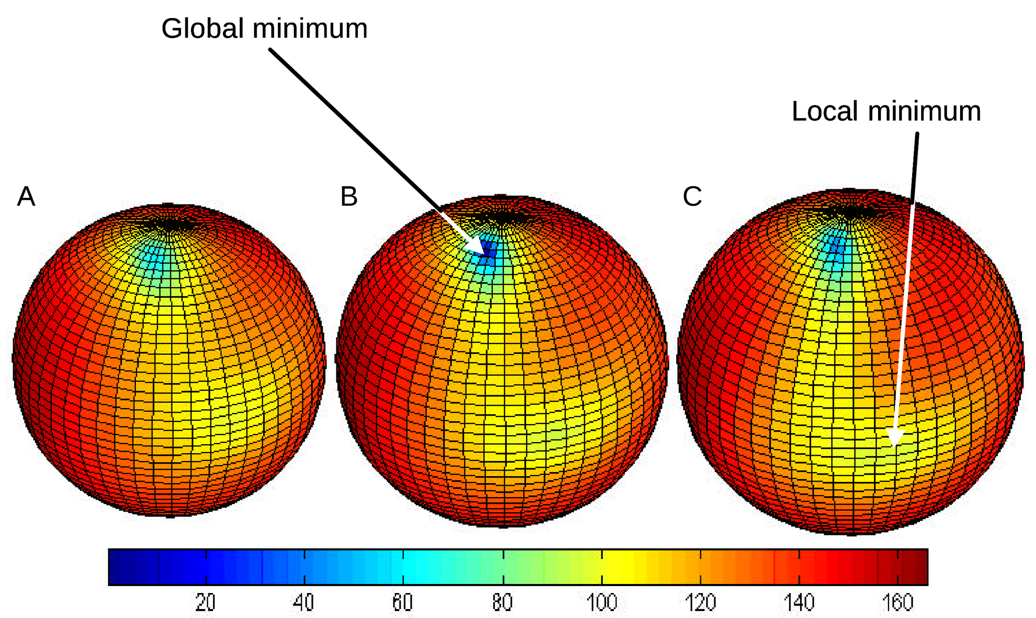
Graphical Representation of the JACOBI MINIMIZATION

Figure 1. Simultaneous Jacobi minimization of two 3 x 3 matrices, A_1 and A_2. Multiple searches are necessary to ensure that the global minimum for R_{HD} has been found. Solid points denote the first Jacobi rotation (applied along the 2,1 axis pair) in the repeating sequence: $J_{2,1}$, $J_{3,2}$, $J_{3,1}$. Red points denote Jacobi rotations that did not lead downhill (and therefore were not applied).



Global Search in a 3-D ROTATION SPACE

Figure 2. In three dimensions a global search for the triangularizing matrix is practical. A three-dimensional rotation space for matrices A_1 and A_2 of figure 1 is represented here as concentric spherical shells. For any three-dimensional rotation, we determine the rotation angle θ and the rotation axis; the rotation angle determines the radius of the spherical shell, and the axis determines the position on that shell. The colors on the spherical shell correspond to the values of the residuals R_{HD} induced by application of the corresponding rotation. The deep blue-colored area in B (θ = 1.3 rad) corresponds to the global minimum. There is a light yellow-green local minimum nearby, centered on shell C (θ = 1.4 rad) but also visible on shell B (θ = 1.3 rad). There is a saddle separating these minima, and descent methods may stabilize on either side of this saddle. All features are visible but less prominent on shell A (θ = 1.1 rad).



APPLICATION TO ICTAL EEG

Although the ictal electroencephalographic (EEG) records of temporal lobe and absence (petit mal) seizure patients appear quite distinct, similarity among their clinical features suggest shared underlying mechanisms. The non-linear autoregressive analysis (NLAR) introduced by Schiff et al. (1999) provided support for this idea. NLAR "fingerprints" obtained from absence EEG traces and those obtained from the PCs derived from temporal lobe EEG records shared evidence of nonlinear interactions at long time lags (ca. 90, 150 msec). However, the PCs that showed these interactions accounted for only a small amount of the variance in the data (less than 12%). We applied hierarchical decomposition to these records to determine the dynamical relationship of the sources of nonlinearity to the overall EEG record.

Quick Tour of the NLAR ANALYSIS

The near-Gaussian character typical of background EEG activity (Elul 1969) is not maintained during an epileptic seizure. Linear autoregressive (LAR) models are therefore insufficient to account for the qualitative features of an ictal discharge. The NLAR analysis augments an LAR model by evaluating the importance of individual non-linear terms in signal prediction.

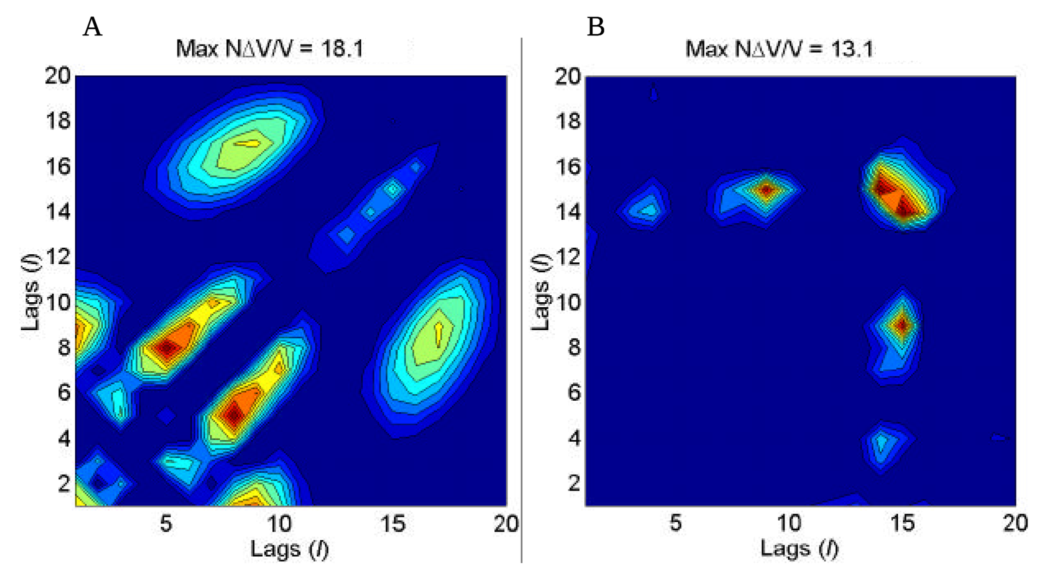
- An NLAR model for an EEG trace is achieved by minimizing the residuals R_{NLAR}, a process analogous to solving the Yule-Walker equations (Yule 1927; Gersh and Yonemoto 1977).
  - $y_n = R_n - m_0 - \sum_{l=1}^{s} b_l y_{n-l} - c_{l,m} y_{n-l} y_{n-m}$
    - b are linear coefficients, c are non-linear coefficients

The NLAR fingerprint is obtained by constructing a contour plot of the $N\Delta V/V$ for each non-linear coefficient $c_{l,m}$, added individually to the autoregressive model above. V is the unexplained variance, and ΔV is the improvement in the model associated with a single term $c_{l,m}$ (For further details on NLAR analysis see, Schiff et al. 1995a; Schiff et al. 1995b.)

- Adding a new model term always decreases residual variance, but at the "cost" of increasing the model's dimension. Victor and Canel (1992) extended the Akaike criterion (AIC) to NLAR models containing a single non-linear term, as a means of statistical justification.
  - $N\Delta V/V > 2$ corresponds to the AIC, and $N\Delta V/V > 4$ implies the more strict condition $P < 0.05$, i.e. that the improvement in the model associated with $c_{l,m}$ is greater than expected from chance.

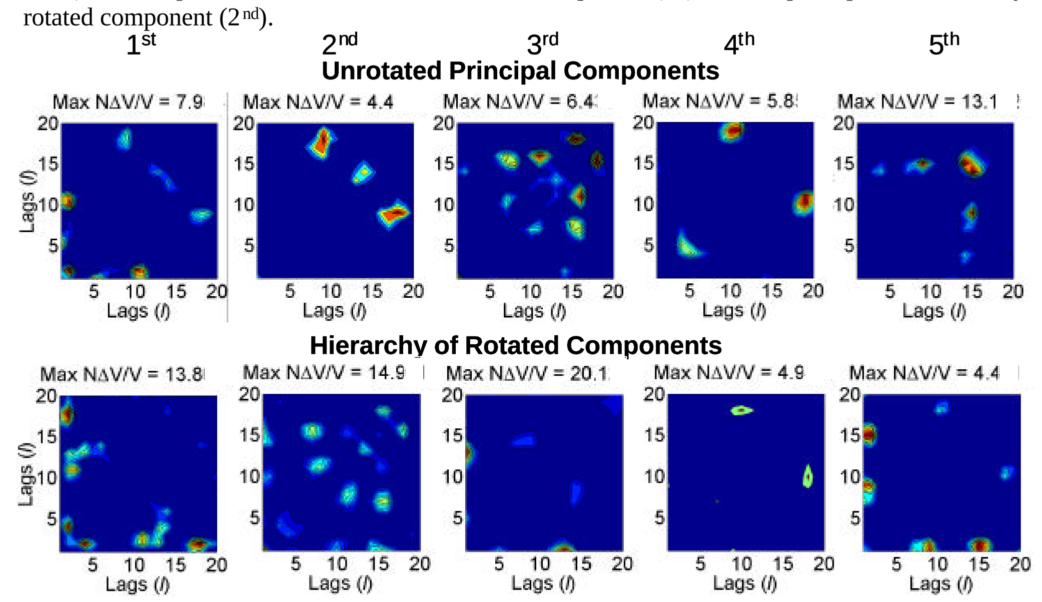
THE NLAR FINGERPRINT

Figure 3. Similarities between NLAR fingerprints from (A) absence epilepsy and (B) the 5th PC from temporal lobe epilepsy (patient 1). Note the peaks in the region of 9, 15 lags (corresponding to 90, 150 msec), and 15, 15 lags (corresponding to 150, 150 msec). Maximum $N\Delta V/V$ values are quite large for both fingerprints. Figures adapted from Schiff et al. (1995a) and (1999).



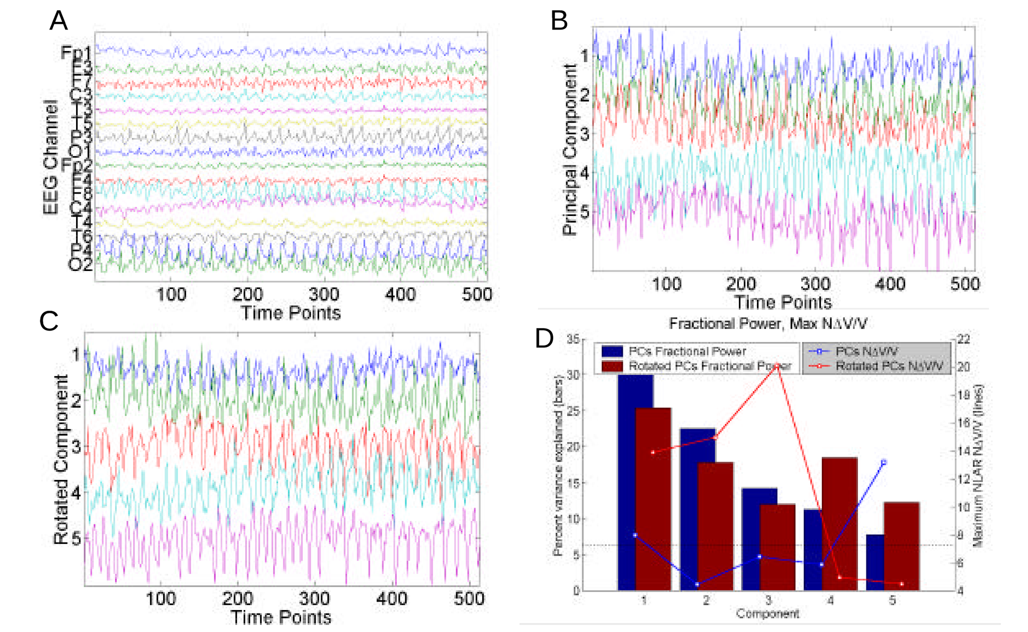
PATIENT 1 NLAR FINGERPRINTS

Figure 5. Comparison between the temporal lobe ictal NLAR fingerprints for the PCs and the rotated components. Note the maintenance of important peaks around 9, 15 lags (90, 150 msec), 15, 15 lags (150, 150 msec), and the exposure of absence-like peaks near 1, 10 lags (10, 100 msec) that are present in the autonomous rotated component (1st) and the principal hierarchically rotated component (2nd).



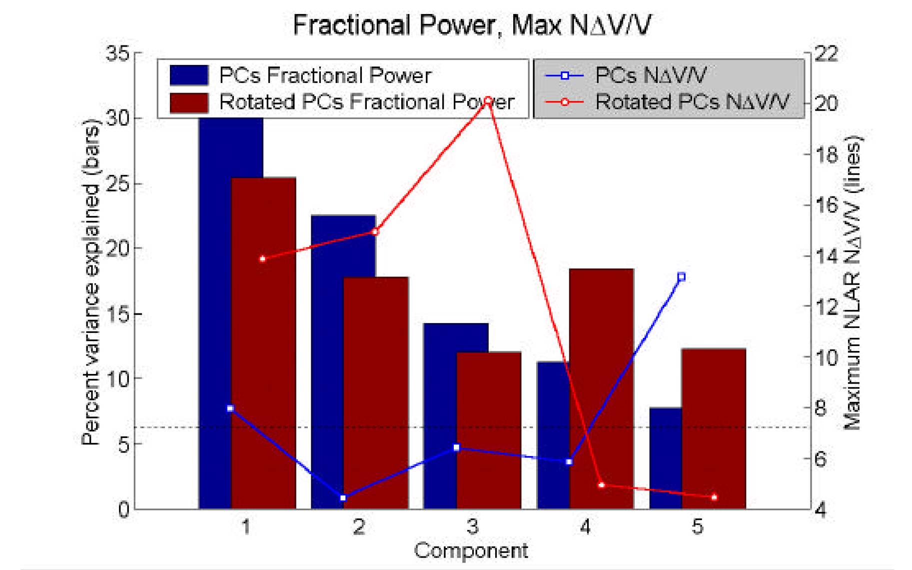
METHOD OVERVIEW

Figure 4. Multichannel data analysis. The PCs (B) derived from temporal lobe epilepsy (patient 1) EEG (A) are rotated according to their MLAR derived hierarchical structure (C). The NLAR fingerprints for all components are generated and summarized by their maximum $N\Delta V/V$ (D).



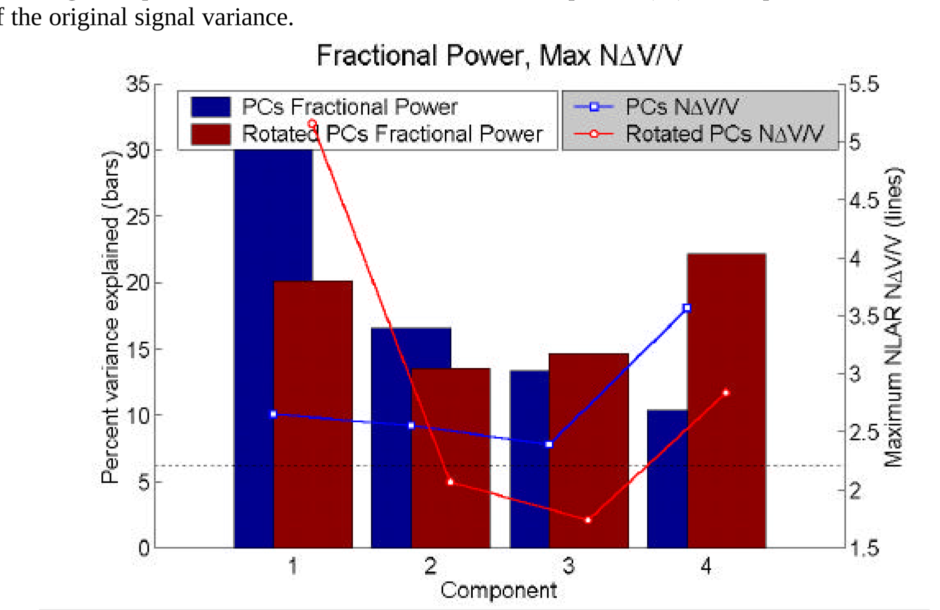
PATIENT 1, SEIZURE 1 SUMMARY

Figure 6. The variance and maximum $N\Delta V/V$ for all components in the multichannel data analysis of temporal lobe epilepsy. Hierarchical decomposition has concentrated the non-linear signals in the first three rotated components. The top three hierarchically rotated components account for greater than 55% of the variance and exhibits a strong non-linear signal dependence.



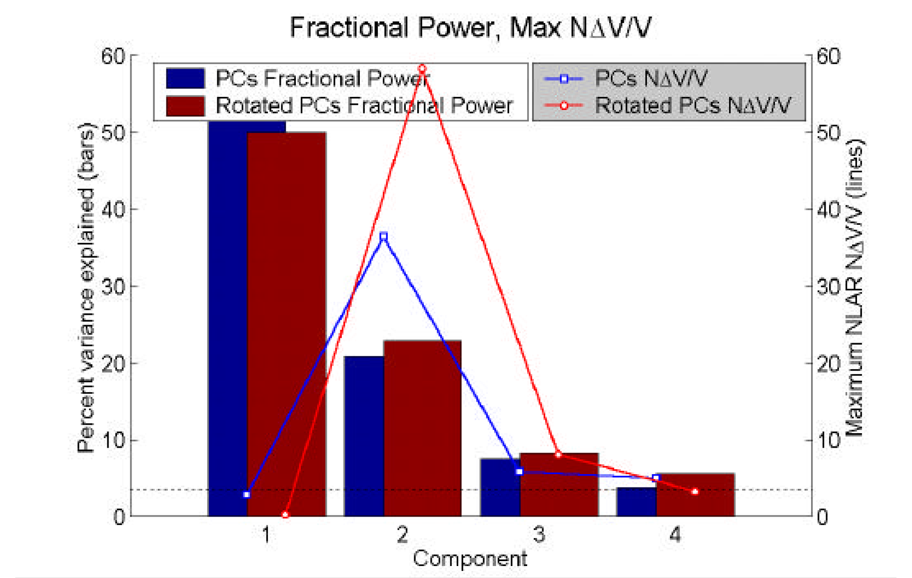
PATIENT 1, SEIZURE 2 SUMMARY

Figure 7. The variance and maximum $N\Delta V/V$ for all components in the multichannel data analysis of temporal lobe epilepsy. Hierarchical decomposition has exposed a significant non-linear signal dependence in the autonomous rotated component (1st) and explains about one fifth of the original signal variance.



PATIENT 2, SEIZURE 1 SUMMARY

Figure 8. The variance and maximum $N\Delta V/V$ for all components in the multichannel data analysis of temporal lobe epilepsy. Hierarchical decomposition has intensified the non-linear signal dependence in the second rotated component, which concurrently is found to account for an increased amount of the variance (ca. 23%).



CONCLUSIONS

- Hierarchical decomposition (HD) exploits the inherent relationships among the numerous channels of data available in a spatiotemporal record, and separates autonomous and hierarchically dependent components of the signal. The results of the preceding multivariate spatiotemporal data analysis suggest that the combination of PCA and HD may provide a method for the resolution of underlying signal generators, thus permitting improved signal characterization.
- With respect to the analysis of seizure EEG, this method has allowed us to resolve a hierarchical relationship among the intrinsic neural "generators". This reveals similar dynamics between the driving nonlinearities that contribute to temporal lobe epilepsy EEG, and those apparent in absence seizure EEG – even though the driving nonlinearities may sometimes constitute only a small portion of the variance.
- This finding provides strength for the suggestion by Schiff et al. (1999) that some patients with temporal lobe epilepsy may share common neural circuit disturbances with those of absence epilepsy.

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